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Mechanical Knowledge and Pompeian Balances

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MECHANICAL KNOWLEDGE AND POMPEIAN BALANCES

Peter Damerow, Jürgen Renn, Simone Rieger, Paul Weinig

1. THE INVENTION OF BALANCES

The balances of Pompeii may appear to be a rather insignificant and even somewhat strange object of research for historians of science. One would not, in particular, expect to find significant insights into the history of mechanical knowledge by inspecting such balances. In this paper it will become clear, however, that these seemingly trivial objects of everyday life in a Roman city pose challenging problems for a history of knowledge. The following text reports about a preliminary investigation of these balances,¹ pursued in the context of a project on the history of mechanical knowledge at the Max Planck Institute for the History of Science in Berlin. In the course of this project such balances are analyzed from the specific perspective of the mechanical knowledge required for their invention, production, and usage, a perspective that is usually not applied to the study of balances in the history of technology. It has turned out that practical knowledge about balances provided the foundation for the theoretical texts on me-

¹ On Roman balances and weights in general see Binsfeld 1990; Franken 1993; Gamurrini 1889; Grönke and Weinlich 1992; Ibel 1908; Jenemann 1989; Jenemann 1994; Knorr 1982, Appendix A; Lazzarini 1948; Paret 1939; Pontiroli 1990; Visy 1994. Some of the balances from Pompeii have been the subject of specialized studies to which this contribution is much indebted (see Della Corte 1912; Di Pasquale 1999; Jenemann 1992; Jenemann 1994). It must, however, be pointed out that the total collection of Pompeian balances and weights has so far never been the object of a systematic investigation. Fortuitous circumstances directed our attention to the fact that this collection comprises a much greater number of balances and weights, partly in damaged condition, than suggested by the available studies. This paper is based on a preliminary survey of the entire collection, prepared during a two-day research stay in Naples and Pompeii, and on the occasion of the "Homo Faber" exhibition on science and technology in Pompeii, displayed in Naples and Munich. The first results reported here show that the collection, because of its historical and geographical coherence, is of unique importance for the reconstruction of the roots of theoretical mechanics in the knowledge of the artisans of ancient cities. It will become clear, in particular, that this collection urgently deserves a more exhaustive analysis. We would like to thank Pietro Giovanni Guzzo and Annamaria Ciarallo of the Sopraintendenza Archeologica di Pompei (SAP), Paolo Galluzzi and Giovanni di Pasquale of the Istituto e Museo di Storia della Scienza (IMSS), Florence, Walther Rathjen and Manfred Spachtholz of the Deutsches Museum, München, as well as the staff of the Museo Archeologico Nazionale (MAN) in Naples for their generous support of our preliminary survey of Pompeian balances and weights. Furthermore, we thank Mohammed Abattouy for sharing with us his translations of Arabic manuscripts, Direttore Papa for extensive demonstrations of the production process of balances of steelyard type in his factory in Fisciano, and the Chinese artisans who allowed us to observe and learn about their traditional techniques of balance production.

chanics from Antiquity and the Middle Ages. The analysis of this text tradition has revealed that their key topic is a mental model of the transformation of forces, based on identifying the lever with a balance with unequal arms.

1.1. EQUAL ARMS BALANCES

The balance probably was invented in the first half of the 3rd millennium B.C. In Egypt the earliest depictions of balances date from the Old Kingdom. In Mesopotamia no early depictions of balances have survived, but the use of balances is well attested to by weight measures occurring perpetually in the administrative documents from the early-dynastic Fara period onwards, that is, since around 2700 B.C. In Mesopotamia, even a terminus a quo for the introduction of the balance is provided by the earlier corpus of proto-cuneiform administrative documents dating back to the turn of the 4th to the 3rd millennium B.C. The absence of weight measures in these documents shows that the balance was not yet known or, at least, not yet used in the same way as only a few hundred years later.

The invention of the balance had significant conceptual consequences. The systems of measures and quantifications which were used at the time of the emergence of state organizations to control social and economical exchange originated predominantly from natural units. Length measures, for instance, were derived from body parts like the foot and the cubit. Capacity measures were derived from containers such as cups and vessels used in everyday life. Weight measures, on the other hand, were from the very beginning based on an invented tool devoted exclusively to the purpose of measuring weights. Thus, the concept of weight was from the very beginning of its quantitative use intimately linked to the development of the balance.

1.2. THE INVENTION OF BALANCES WITH UNEQUAL ARMS AND THE DISCOVERY OF THE LAW OF THE LEVER—A HISTORICAL COINCIDENCE?

This explains a remarkable fact. For nearly 2500 years there was no substantial change in the technique of weighing and, correspondingly, there was no substantial change in the concept of weight. After this vast period of stagnation, however, two things happened virtually at the same time. Balances with unequal arms were invented, and, based on the discovery of the law of the lever, the science of mechanics was created and the concept of weight was transformed into one of its basic categories.

The later designation of the new bit of knowledge which represents this "discovery" as the "law of the lever" is misleading. In view of the context of the texts which document its discovery this law should rather be read as the "law of the balance with unequal arms." There can, at least, be no doubt that the coincidence of the invention of balances with unequal arms and the emergence of a science of mechanics based on the "law of the lever" was not accidental. The two events must, in fact, have been closely related to each other.

There are different reasons for constructing balances with unequal arms and different ways to realize them. The main reason for constructing such balances is that the use of standard weights can be substituted by simply varying the length of the arms of the balance along a calibrated scale. In principle, this can be realized either by a variable fulcrum, the so-called Bismar type of balance, or by a variable position of a counterpoise, the Roman steelyard, or by a variable position of the load to be weighed, a type not used very much and therefore without a specific designation.² Furthermore, the counterpoise of the second type has often been used in combination with a balance with equal arms to determine fractions of a weight standard.

To answer the question of what precisely was the nature of the relation between the invention of such balances and the origin of mechanics as a science was the primary motivation for the present investigation of balances, some results of which are reported in the following. A simple answer to this question is often tacitly assumed and sometimes even explicitly asserted. The invention of balances with unequal arms is considered as an immediate application of the law of the lever in the sequel of its discovery.³ This assumption, however, is obviously wrong as will become clear below.⁴ But if this assumption is wrong and balances with unequal arms were already in use before the law of the lever was known, another question arises. If the discovery of the law of the lever was a consequence and not a precondition of the invention of such balances, what then was the knowledge the invention of these balances was based on?

² For a classification of balances with unequal arms see Jenemann 1989, pp. 320f.; Jenemann 1994, pp. 200-204. ³ The balance with unequal arms is usually discussed as an application of the law of the lever without clarifying whether or not this implies that its invention preceded the discovery of this law, see e.g. Paret 1939, pp. 73-75; Garbsch 1992, p. 231; Jenemann 1995, pp. 145f. The obliviousness with which the relation between theoretical and practical knowledge is usually treated may be illustrated by the way in which Jenemann dealt with this problem in Jenemann 1989, pp. 322f. and in Jenemann 1994, pp. 201f., in particular footnote 17. Whereas in his earlier publication he argues that Archimedes must have found the law of the lever by means of a balance with unequal arms, he considers in his later publication the possibility that one of the disciples of Archimedes or even Archimedes himself might have invented the so-called Roman balance as an application of work on the law of the lever. Jenemann does not consider it worthwhile to explain the contradiction between both statements and to clarify his own opinion about the role which practical experience with balances played in the discovery of the law of the lever.

⁴ Various authors have come to the conclusion that the balance with unequal arms must have been invented already when the law of the lever was established, see e.g. Knorr 1982, p. 134.

2. THE ORIGINS OF THEORETICAL MECHANICS

The earliest source documenting the emergence of a science of mechanics also provides us with information about the invention of balances with unequal arms. It is a treatise with the title "Problems of Mechanics," traditionally ascribed to Aristotle and originating, in any case, approximately at the end of the third century B.C.⁵ Concerning this text we will raise here two questions:

- First, were balances with unequal arms invented before this treatise was composed?
- Second, did the author of the treatise already know the law of the lever?

The "Problems of Mechanics" consists essentially of 35 questions and their answers, most of which follow precisely the same pattern of argument. First, a problem is posed beginning with a question such as: "Why is it that [...]," followed by the description of a device or technique that makes it possible to overcome a great force by a smaller one. Second, certain elements of the mechanical arrangements constructed for such a purpose are identified with the essential parts of a lever, that is with the bar, the fulcrum, the motive force, and the weight to be moved. Third, the application of a principle which was considered as characteristic of both the lever and the balance and which is, as a rule, explicitly stated before it is applied and, in general, the result is stated once more in terms of the posed problem. The principle says:⁶

Moved by the same force, that part of the radius of a circle which is farthest from the centre moves quicker than the smaller radius which is close to the centre.

The first problem plays a special role. In the course of the discussion of this problem the principle is not simply stated but justified by an extensive proof which attempts to show that the principle follows as a consequence from Aristotelian physics. This proof is based on the interpretation of the lever as performing a circular motion in the same way as a balance. When the principle is applied, the pattern of argument relates the transformation of forces by mechanical devices to experiences which can be gained from balances when the length of the arms are varied. We could say, the balance, identified with the lever, is used as a mental model in order to explain the transformations of forces which were characteristic of ancient mechanical technol-

⁵ In the 19th century it became customary to question the ascription to Aristotle. According to our own investigation (to be published) it has, however, turned out that there is strong evidence for the traditional position.

⁶ This principle is explicitly formulated in the "Problems of Mechanics" more than fifteen times. See, e.g. Aristotle 1980, p. 337: "The origin of this is the question why that part of the radius of a circle which is farthest from the centre moves quicker than the smaller radius which is close to the centre, and is moved by the same force."

ogy. What is explained by the "Mechanical Problems" is the basic knowledge of practitioners. The lever-balance model serves as a fundamental theoretical tool for integrating such knowledge into Aristotelian physics.

This purpose of the treatise is explicitly dealt with in its introductory part before the first problem is discussed. The treatise begins with a statement of the general problem of combining Aristotelian physics with the knowledge on which the transformation of forces in mechanical technology is based:⁷

"Remarkable things occur in accordance with nature, the cause of which is unknown, and others occur contrary to nature, which are produced by skill for the benefit of mankind."

In the following, the pattern of argument used in the treatise to make Aristotelian physics compatible with mechanical technology is characterized by pointing out the role of the balance.⁸

"The facts about the balance depend upon the circle, and those about the lever upon the balance, while nearly all the other problems of mechanical movement can depend upon the lever."

This pattern of argument which essentially explains the effect of the lever by identifying its motion with the circular motion of the beam of a balance and the different effects of weights in different distances from the center provide a strong indication that the author of the treatise was well acquainted with experience gained by the use of balances with unequal arms.

2.1. THE BISMAR IN THE "PROBLEMS OF MECHANICS"

The familiarity of the author of the "Problems of Mechanics" with balances with unequal arms is confirmed by the fact that one of the problems⁹ explicitly deals with such a balance. The problem raised is why is it possible that by means of a balance a heavy piece of meat can be compensated for by only a small counter weight. In the course of the discussion of this problem the author gives a description of the balance he had in mind. Although it is not easy to infer from the text how this balance was constructed—it seems to be a type of Bismar with varying sus-

⁷ Aristotle 1980, p. 331.

⁸ Aristotle 1980, p. 335.

⁹ Problem 20; see Aristotle 1980, pp. 375-377.

pension—there can be no doubt that the author is speaking about a type of balance for weighing meat, well-known at his time, which used the transformation of forces by a balance beam with unequal arms.

An independent confirmation of this conclusion is provided by poetry written some one hundred years earlier. Aristophanes in his comedy "Peace"¹⁰ let Trygaeus mock an instrument maker by saying that he should pour lead into his trumpet and fix a dish at the other end in order to use it for weighing figs for his slaves in the fields. This joke makes sense if it is assumed that the audience of Aristophanes was well acquainted with a Bismar type of balance with a fixed weight at one end, a scale pan at the other end, and a movable suspension in its middle. That is, at the time of Aristophanes the use of the Bismar must have been already so well established that he could use it in a joke without any further explanation.

Summing up the evidence concerning the question of whether balances with unequal arms were already invented when the author of the "Problems of Mechanics" composed his treatise, we can conclude that balances with unequal arms were already well established at this time, whatever their precise construction may have been. Thus, he could derive from this instrument a general principle explaining the transformation of forces by means of mechanical devices.

2.2. THE "LAW OF THE LEVER" IN THE "PROBLEMS OF MECHANICS"

The natural place to search for an answer to the second question of whether or not the author of the "Mechanical Problems" knew the law of the lever is Problem 3 of the treatise which deals explicitly with the lever.¹¹ In accordance with the general pattern of argument, the problem is raised:

"Why is it that small powers ($\delta \nu \nu \dot{\alpha} \mu \epsilon \iota c$) move big loads ($\beta \dot{\alpha} \rho o c$) by the lever (μοχλός) [...]?"

As usual in this treatise the answer is based on the main principle that small forces can move great weights since:

"[...] under the same (moving) load the greater radius ($\dot{\epsilon}\kappa \tau \sigma \hat{v} \kappa \dot{\epsilon} \nu \tau \rho \sigma v$) moves faster [...]"

The conclusion is finally drawn:

¹⁰ Aristophanes 1938, 1240-1249.
¹¹ For the Greek text see Aristotle 1980, p. 353. New English translation by the authors.

"[...] that by the same force $(lc\chi \dot{v}c)$ the moving will change $(\mu \epsilon \theta l c \tau \eta \mu l)$ more the more away (it is) from the hypomochlion."

In the course of this argument,¹² the lever is identified, point by point, with a balance of unequal arms:

"Is it that the lever is the reason, [being] a balance $(\zeta \nu \gamma \delta \nu)$ having the suspension $(c\pi a\rho \tau i o\nu)$ from below and [being] divided in unequal parts? Namely, the hypomochlion substitutes the suspension; for both of these remain the same ones (= remain stationary), just-as the centre."

Once this identification of lever and balance with unequal arms has been established, and after the main principle has been invoked, all of a sudden the law of the lever appears:

"And those concerning the lever are three: the hypomochlion—suspension and centre—and two loads, namely the moving and the moved. Now (as) the moved load to the moving, (so) the length to the length inversely."

The last statement neither directly follows from the preceding argument nor is actually used at this or any other point in the treatise, not even when the balance with unequal arms is at issue.

This puzzling appearance of the law of the lever "out of the blue" suggests that it may represent a commentary rather than a logical step of the conclusion to be reached. It even raises the possibility that the corresponding passage is actually due to an insertion into the text made by a later copyist. In any case, for the author of the "Mechanical Problems" the law of the lever was surely not part of the foundation on which he built his theory. If he realized this law at all, he certainly failed to recognize its significance. It could hardly have meant more for him than a minor consequence of his identification of the lever with a balance. It was a "spin-off," so to say, of his knowledge of the balance with unequal arms, worth at most a passing remark.

2.3. PRACTICAL KNOWLEDGE IN THEORETICAL TREATISES

But if the balance with unequal arms was invented a considerable time before the law of the lever was discovered, the problem is not only to determine on the basis of what knowledge this invention was possible but also to answer the question of how practitioners were able to consistently build, use and improve such balances without knowing the law explaining their function.

¹² For the Greek text see Aristotle 1980, p. 353. New English translation by the authors.

Unfortunately it is very difficult to gain information about such knowledge which is usually never written down and only survives in the artifacts that have been preserved. Do the treatises of later mechanics harbor more of this information than we can extract from the Aristotelian "Problems of Mechanics"?

The contrary is the case. Compared with treatises written after the discovery of the law of the lever, this first treatise on mechanics-although focussed on the theoretical reflection of mechanical technology-still carries traces of the knowledge of the practitioners who used this technology. In the sequel of further elaboration of such theoretical reflections, Euclid conceived in his concise treatise on the balance highly abstract foundations for deriving the law of the lever.¹³ Archimedes in his work on the equilibrium of planes ascribed his ingenuous reduction of the law of the lever to elementary assets of intuitive physics such as the proposition that weights which balance at equal distances are equal.¹⁴ Both treatises, which later became cornerstones of the development of mechanics, no longer even mention the various tools and techniques of practical mechanics which the author of the Aristotelian "Problems of Mechanics" took as his starting point for the development of the first mechanical principles. The rich experiences of the practitioners transmitted only by participation and oral communication gave birth to deductive systems of mechanics transmitted only by a literary culture, but seemingly were no longer worthy of notice in the shadows of the crystal palaces erected by these systems.

3. RIDDLES POSED BY POMPEIAN BALANCES

Thus the question of how a technology of balances presupposing the law of the lever could be developed by these practitioners before the law itself was discovered necessarily leads to an investigation of the material remains of this technology. This was also the motivation for launching the above mentioned research project at the Max Planck Institute for the History of Science, which attempts to extract from archeological remains as well as from surviving traditions of craftsmanship insights into the practitioners' knowledge connected with the production and use of balances with unequal arms. Here we want to illustrate by way of some samples from Pompeii how such insights can be achieved.

¹³ See Woepcke 1851.¹⁴ See Archimedes 1953.

There are essentially two types of data which we systematically collected in Naples and Pompeii. First, the precise values of standard weights, counterpoises etc. were measured for more than 200 excavated objects accessible to us in Naples and Pompeii. Second, data about all accessible balances were recorded, including detailed measurements of the geometry of steel-yard type balances.

3.1. WEIGHTS

					Calculated	
Size	Label	Location	Inventory number	Weight	weight unit	Deviation
1 uncia		Pompeii	SAP 20350	27.0 g	27.0 g	0.4%
	1 point	Naples	Unreadable	31.0 g	31.0 g	15.2%
		Naples	MAN 7371	26.5 g	26.5 g	- 1.5%
3 unciae	3 points	Pompeii	SAP 11900 A	83.6 g	27.9 g	3.6%
	3 points	Naples	MAN 12198 (?)	79.0 g	26.3 g	- 2.1%
4 unciae	4 points	Pompeii	SAP 14482	107 g	26.8 g	- 0.6%
		Naples	MAN 1111?	106 g	26.5 g	- 1.5%
6 unciae	S	Pompeii	SAP 20351	157 g	26.2 g	- 2.7%
	S	Naples	None	163 g	27.2 g	1.0%
1 libra	Destroyed	Pompeii	SAP 3305 C	319 g	319 g	- 1.2%
	None	Naples	MAN 74218	317 g	317 g	- 1.9%
	Ι	Naples	MAN 11.1885	325 g	325 g	0.6%
	Ι	Pompeii	SAP 2853	318 g	318 g	- 1.5%
2 librae	II	Naples	Unreadable	615 g	307 g	-4.8%
	SEX II TRA	Pompeii	SAP 12907	637 g	318 g	- 1.4%
	II	Naples	MAN 74204	648 g	324 g	0.3%
3 librae	V III ?	Naples	MAN 64117 (?)	1030 g	343 g	6.3%
	Unreadable	Naples	Unreadable	1070 g	357 g	10.4%
	III	Naples	MAN 74203	962 g	321 g	- 0.7%
	Unreadable	Naples	MAN 74197	930 g	310 g	- 4.0%
5 librae	V	Pompeii	SAP 2504	1605 g	321 g	-0.6%
	V	Naples	MAN 74190	1610 g	322 g	-0.3%
10 librae	Х	Pompeii	SAP 20349	3562 g	356 g	10.3%
	Х	Pompeii	SAP 2503	3217 g	322 g	- 0.4%
	Х	Naples	MAN 74185	3213 g	321 g	- 0.5%
	Х	Naples	Unreadable	3230 g	323 g	0.0%
	Х	Naples	MAN 74180	3385 g	339 g	4.8%
	Х	Naples	Unreadable	3208 g	321 g	- 0.7%
20 libra	XX SOC CRT	Pompeii	SAP 3918	6300 g	315 g	- 2.5%
	XX	Naples	Unreadable	6250 g	313 g	- 3.3%

Table 1: STANDARD WEIGHTS MADE OF BASALT

As far as the weights are concerned we will restrict ourselves here to mentioning only two results. The *first result* concerns the way how conclusions are usually drawn from measures of standard weights which have survived. Mostly, such data taken from standard weights that come from different locations and times are used to precisely determine values of the historical weight units, often recorded up to 5 digits. Deviations are explained as changes of certain units which are attributed to specific historical events. The Pompeian weights, however, all come from the same place and the same time. Therefore they make it possible to evaluate the implicit assumption of this methodology that already in ancient times weights were highly standardized. Our data, however, show that this was not the case.¹⁵

Table 1 contains only standard weights which are made of basalt¹⁶ and show no damages so that changes of their weight since the time they were used are practically excluded. The table shows that these weights consistently belong to one and the same weight system based on the libra consisting of twelve unciae.¹⁷ The first column of the table gives the sizes of the standard weights in ancient units (first column) which is sometimes indicated by inscriptions (second column). The standard weights are identified by the place where they are kept (third column) together with their inventory number (fourth column). Further the table provides information about the real weights of the preserved standard weights (fifth column). The variation of the investigated standard weights is then indicated by the sizes of the units calculated from the measured weights (sixth column), and by the deviations of these figures from a Pompeian libra of 323 g or its twelfth part respectively (seventh column). But within this system the degree of standardization is rather low. The individual weights differ up to 15 percent from each other so that we could not chose a reference value for our investigation without some anachronistic presupposition of a "true" value.

¹⁵ We do not claim that there were no attempts in the Roman empire—as in other ancient states—to standardize weights as well as other measures. It is well known that there existed so-called "pondera publica" kept in "ponderaria" which served to control the weights used in the market places. In the following it will become clear, however, that according to the data fromPompeii this system did not work well.

however, that according to the data fromPompeii this system did not work well.
 ¹⁶ An additional standard weight of 5 librae kept in Naples (inventory number MAN 74192) which is also made of basalt could not be investigated since it was and still is part of the ongoing "Homo Faber" exhibition.

¹⁷ See Hultsch 1882, in particular p. 161. Hultsch argues (pp. 155ff.) that there existed precise sizes of the libra (327.45 g) and the uncia (27.288 g) which remained constant over a long time period. According to him, the observable deviations of coins and preserved standard weights from the value given by him are due to errors or special local traditions. In spite of such deviations determined from archeological finds, the precise value given by Hultsch is generally accepted until today, see, e.g. Binsfeld 1990 and the article "Gewichte" in the standard reference work for knowledge of the ancient world Cancik and Schneider 1998, cols. 1050-1056, in particular cols. 1053-1056 on Roman standard weights. An exceptional archeological finding contradicting this value was published in Lazzarini 1908. Excavations at Palestrina have revealed a nearly complete set of weights found near a public administrative building in which a "ponderarium" may have been located. Lazzarini interprets the set of weights as a set of "pondera publica" the existence of which is known from literary sources. The weights of these objects have been precisely determined in the Laboratorio Centrale Metrico di Roma. The average size of the libra represented by this set of weights turned out to be 323.220 g, differing considerably from the generally accepted value of Hultsch. We are using here a value of 323 g close to Lazzarini's which is in good agreement with the values determined for the Pompeian weights. We do not claim that this value of 323 g serving here as reference value was any precise norm in the Roman empire. As long as no comparison between "pondera publica" of different locations and times is possible, nothing can be said about the existence or non-existence of precise weight standards at that time. It is, however, a remarkable fact that the deviations of the weights from their average published by Lazzarini are much smaller (less than 2%) than those of the Pompeian collection analyzed here. This observation strongly supports Lazzarini's identification of the finding as representing in fact "pondera publica."

The second result of our analysis of weights from Pompeii points to the existence of a second, independent weight system. There are three weights¹⁸ also made of stone not contained in the table inscribed with the Roman numerals X and XX, but the size of the represented unit is not that of the libra, that is about 320 grams, but only about 8.5 grams, that is a figure which has no simple numerical relation to the weight of the libra. The existence of such an independent second system is further confirmed by standard weights which were not made of stone. In particular, there are two very precise weights in standard form inscribed with four dots arranged in a square which represent the unit of this system.¹⁹

3.2. BALANCES

The total number of balances surveyed is 50. The majority of these balances, 27 finds, are balances with equal arms. It is a remarkable fact that, wherever the state of preservation permitted a judgement, all of them turned out to carry a scale with 10 points dividing one arm into 12 parts, pointing to their use together with a counterpoise which in several cases had, in fact, been found together with them.²⁰ A plausible explanation would be that the movable counterpoise served as a means to determine fractions of the weight unit.²¹ Given the average sizes of the balances we inspected, the markings of the scales would thus naturally correspond to multiples of the dimidiasextula which is the twelfth part of a uncia.²² However, this interpretation meets great difficulties when confronted with a broader survey of the extant archeological findings. A great number of objects resembling such counterpoises have been excavated, albeit mostly without being directly associated with a balance. These counterpoises differ in weight from standard units such as the uncia. Our preliminary survey furthermore strongly suggests that they were produced to represent the multiples of the weight unit of about 8.5 grams mentioned above, hinting at their use also for quite different, more elaborate purposes.²³ While the pursuit of this question evidently requires a more systematic analysis of these finds within their archeological context, it seems clear, in any case, that balances with scales and a complex system of counterweights embody mechanical knowledge beyond that involved in ordinary equal arm balances.

¹⁸ All three weights are kept in Pompeii. There inventory numbers are SAP 6888 B (weight: 85.4 g; inscription: X), SAP 3355 (weight: 170 g; inscription: XX), and SAP 53510 (weight: 169 g; inscription: XX).

¹⁹ Up to now we have not been able to identify this second Pompeian weight system with one of those reported in the literature.

²⁰ This supports the conclusion drawn by Lazzarini on the basis of his analysis of balances kept in the Museo Nazionale Romano and the Antiquarium Comunale of Rome, see Lazzarini 1948, p. 222. ²¹ See e.g. Jenemann 1985, p. 167; Grönke and Weinlich 1992, pp. 192f.

²² Such balances may, in fact, have existed. For the nearly complete balance SAP 12217 carrying, as usual, a scale subdivided in 12 units, a counterpoise has been preserved with a weight of 29.3 g, differing from an uncia by about 9% only.

The following analysis deals with the 23 unequal arms balances on which the present survey was concentrated. With one exception they are of the steelyard type. Not all of them were, however, accessible to us because they were part of an exhibition. Thus, 19 balances with unequal arms were actually included in our present investigation.²⁴

3.3. THE "CASSEROLE BISMAR"

If we can trust the hints given by authors such as Aristophanes and by the author of the "Problems of Mechanics," balances of the Bismar type belong to the earliest types of balances with unequal arms. Our discussion of balances with unequal arms will first deal with a remarkable exemplar of this type that has been found in Pompeii.

Its unusual "home-made" construction makes this balance a unique artifact. It is, in fact, quite different from all the other balances in the collections in Naples and Pompeii which are obviously professionally produced. Here, someone must have misused as a balance what originally was a very common kitchen casserole as hundreds of them have been found in Pompeii (see figure 1). A slit has been cut into the handle in order to hold a movable suspension. At the end of the handle a device for hanging the load must have been fixed which now, however, has been lost. Along the slit a scale has been incised ranging from 1 to 12 librae. By moving the suspension in the slit the load could be balanced against the heavy casserole cup in order to determine the weight on the scale.

²³ Although the existence of such counterpoises used with balances with equal arms is often mentioned or depicted in the scholarly literature, the question of the weight of these counterpoises is seldom raised and no weights of the preserved counterpoises are given; see e.g. the catalogue of weights in Franken 1994 which in general provides no weight measures. An exception is Lazzarini 1948, who deals with balances with equal arms and, in this context (pp. 222f., 227f.), explicitly with the sizes of their counterpoises. However, instead of studying the actual weights of archeological finds, he merely inferred the approximate weights of the probably missing counterpoises of the balances he investigated. He estimated the maximal load of the balances and concluded from these estimations the sensitivities of the balances which he interpreted as determining the sizes of the counterpoises. His procedure presupposes, in particular, that the counterpoises must have always represented 12 times a common weight unit, a presupposition actually not borne out by his results. For 14 balances with scales he estimated seven times a counterpoise of a semuncia (the twelfth of which, representing the unit of the scale, is a scripulum), four times a counterpoise of a sicilicus (the twelfth of which is an obulus), two times a counterpoise of a dimidiasextula (the twelfth of which is a siliqua), and one time a counterpoise of a sextula (the twelfth of which are two siliqua). Our preliminary investigation of the actual weights of objects found in Pompeii which may have served as counterpoises of balances with equal arms do not support any of Lazzarini's speculations. While they differ in a similar wide range of weights as Lazzarini's estimates, they do not at all cluster around weight units, let alone those supposed by Lazzarini.

²⁴ The 18 steelyard type balances included have the following inventory numbers: MAN 5749; MAN 74..4; MAN 74030; MAN 74051; MAN 74069; MAN 74076; SAP 1975; SAP 10280; SAP 11093; SAP 11231; SAP 1200; SAP 12934/60; SAP 13417; SAP 13448; SAP 20200; SAP 4037; SAP 6521; SAP 66/29.906; the balance of the Bismar type has the inventory number: MAN 74165.



Figure 1. KITCHEN CASSEROLES



Figure 2. THE "CASSEROLE BISMAR"

Although the function of the casserole to serve as a balance of the Bismar type was recognized early on, there is a long tradition of misunderstandings concerning the question of precisely how the casserole Bismar was used. This confusion reveals that to know the law of the lever seems not to be sufficient to immediately understand the clever idea of the practitioner who built the balance and to reconstruct the knowledge that made it possible. Even now, and not for the first time,²⁵ somebody has attached a completely meaningless counterpoise—a typical counterpoise of a steelyard type balance—to the end of the handle, probably misinterpreting the cup of the casserole as a scale pan.

The implausibility of this reconstruction becomes evident from a comparison of the Pompeian casserole balance with a schematic representation of a Bismar-type balance (see figure 2). A closer inspection of the scale of the casserole Bismar leaves, in fact, no doubt about the true function of the construction. Even the mere direction of the scale on the handle shows that the weight had to be suspended at the end of the handle, rather than placed within the casserole. But the scale of a Bismar has a further peculiarity when compared to that of a steelyard. Due to the fact that the suspension point is moveable, a non-linear scale results on which equal weight differences are represented by shrinking intervals towards the higher end of the scale.²⁶ These intervals converge to zero at a virtual scale mark located at the point from which the load to be weighed is suspended. In fact, at this point a virtually infinite weight could not disturb the equilibrium since it would be placed directly below the suspension. In short, this virtual convergence point of the non-linear scale is determined even by its basic qualitative features, excluding any other interpretation of the casserole Bismar, e.g. as serving to weigh liquids.²⁷

It is remarkable that the non-linear scale on the handle determines the possible constellations to such an extent that its analysis allows us to reconstruct the whole original arrangement. In combination with the weight of the cup serving as counterpoise even the size of the unit of the scale and the weight of the missing scale pan can be calculated from the geometry of the scale. In particular, it is possible to determine the two theoretical ends of the scale, that is the virtual point

²⁵ See Jenemann 1994, p. 223 and, in particular, Jenemann 1992 where the comedy of mistaken attempts to make sense of the casserole Bismar as a balance for weighing liquids is extensively discussed. The erroneous interpretation is given already in a short early publication of 1854 (see Jenemann 1992, p. 525). A depiction of the balance in this publication shows that at that time somebody has fixed already an egg-shaped counterpoise at the end of the casserole handle. Later, the fatal error must have been realized. From replications produced at the end of the 19th century can be inferred that the wrong piece was then detached. Subsequently, however, somebody felt the necessity to attach again a counter weight to the handle of the casserole. This time a nicely shaped head of Mercury, surely belonging to a balance of steelyard type, was chosen as a substitute for the alleged missing counterpoise. The comedy did not even end with Jenemann's publication which convincingly revealed the true function of the casserole Bismar. It was apparently when the current Homo Faber exhibition was displayed in Los Angeles in 1999, that this counterpoise has been permanently fixed by a massive ring. For the time being, the casserole Bismar is exhibited in this fallacious manner.

²⁶ From the law of the lever it follows immediately that, for an ideal Bismar, the sequence of scale intervals is a "harmonic division" of the beam.

²⁷ See e.g. Knorr 1982, Plate 10.

of equilibrium of the balance without load somewhere inside the cup and the point at the end of the handle where the hook for the load had originally been fixed. When the movable suspension in the slit comes close to the latter point, the load compensated by the cup serving as the counterpoise becomes virtually infinite. This follows, of course, from the law of the lever. Only for practical reasons the scale ends at 12 librae, that is at about 4 kilograms.

Using a graphical procedure to fit this theoretical model to the visible scale of the Pompeian Bismar, it has been possible to reconstruct precisely the place of the original hook for the load (see figure 3).²⁸ This reconstruction is confirmed by the fact that exactly at the place of the hy-



Figure 3. SCALE OF THE "CASSEROLE BISMAR"

²⁸ The graphical representation of the harmonic division of the scale used by this procedure and shown in the figure was proposed by an 18th century Swedish author for constructing the Bismar scale, claiming that he was the first to provide such a theoretical construction for this scale which hitherto was allegedly only generated by an empirical procedure, see Polhem (Polhaimer) 1716.

pothetical end of the scale a rivet fixing a now broken ring is visible which originally must have held the hook for the load.²⁹ As it thus turns out, the modern restoration of the casserole Bismar is not only based on a misinterpretation of the cup of the casserole as a scale pan which lead erroneously to attaching to the end of the casserole handle an unrelated steelyard type counterpoise instead of a hook for the load. Moreover, this attachment was put in the wrong place neglecting the still existing remains of the original suspension ring for the now missing hook.

Test measurements with known weights were performed in order to determine the weight units represented by the scale.³⁰ In spite of the fact that the Bismar is heavily corroded and underwent a 'restoration' procedure, these measurements left no doubt that the scale is gauged in libra and that no scale pan of significant weight was attached to the load suspension. Probably, the load suspension merely carried a simple hook, as is well known from many other cases of Roman balances, of steelyard type as well as balances of the Bismar type.

How did the clever inventor of the "Casserole Bismar" determine his scale? The engraved notches of the scale differ from the true values determined by the law of the lever in both directions up to 1.5 millimeters. This error fluctuation indicates that he did not use a theoretical construction as we did here. He must have used instead some kind of empirical procedure for which such fluctuations are typical.³¹ In any case, it must have been one simple enough to encourage even the improvisation of such a balance by abusing a piece of household equipment. Nevertheless, the complicated intrinsic nature of the scale of a Bismar-type balance imposes constraints on its practical realization. Due to its non-linearity, it is not robust against changes of the weights of the elements of the construction. Any modification of the suspension device or of the counterpoise also enforces changes of the scale, even if such modifications are compensated for in such a way as to reestablish the equilibrium of the balance without load. To know such limitations must have been crucial for the practitioners who designed and built Bismartype balances, whereas to know the law of the lever must have been of comparably minor value for them. On the other hand, the Bismar could easily lead to an understanding of how the effect of a weight depends on its distance from the center of its circular motion, exactly as it was re-

²⁹ The graphical procedure applied used the software "Adobe Illustrator" to vary the remaining free parameters under the constraints of the law of the lever and the empirical data embodied by the measurements taken of the Bismar scale. Surprisingly it turned out that the load could not have been fixed at the point where the counterpoise had been erroneously attached. It confirmed the validity of the reconstruction which a high-quality photo we received afterwards revealed that the actual load suspension was located precisely at the point we had determined.

³⁰ mined.
³⁰ These measurements had to be performed without the missing hook, thus necessarily leading to values of the test weights that would be too large by the amount of its weight. The test measurements used the attached counterweight of approximately 900 g and a weight of 500 g, resulting in about 2.9 and 1.7 librae, respectively. Using a value of 323 g for the libra, determined from the extant Pompei basalt weights, these values correspond to 937 g and 549 g, respectively. Given the damaged state of the casserole Bismar, these values within a 10% error range leave no doubt as to the interpretation of the scale as representing librae.

³¹ The errors seem to show a certain periodicity for which we do not have a satisfactory explanation.

flected in the main principle of the ancient author of the "Problems of Mechanics." It was, after all, the simplicity of its construction that may have given this type of balance an important function in the early development of theoretical knowledge in mechanics.

3.4. STEELYARDS

According to the survey of the Pompeian balances, the standard type of balances with unequal arms was not the Bismar but the steelyard (see figure 4). Contrary to the Bismar type, the production of this type of balance required, as will be argued in the following, the knowledge and competence of a skilled artisan.

At first sight, however, a steelyard represents a rather simple device. It has a short arm from the end of which the load is suspended—either by a hook or by means of a scale pan—and a longer arm to which a moveable counterpoise is attached. The longer arm carries a scale indicating the position of the counterweight, from which the weight of the load can be read off when the bal-



Figure 4. THE STEELYARD OR "ROMAN BALANCE"

ance is in equilibrium. The relations between the weight of the load, the weight of the counterpoise, the distance between fulcrum and load suspension, the position of the counterpoise and the length of the unit of the scale are evidently governed by the law of the lever.

The question of how a steelyard is constructed therefore seems to have two alternative but equally straightforward answers. Its various parameters are either calculated and designed according to the law of the lever or empirically determined, if necessary, by trial and error. It therefore comes as no surprise that these two alternatives are precisely the views one encounters in the literature.³²

A closer look at a real steelyard shows, however, that its construction involves more parameters and constraints than those which can be immediately dealt with by the simple law of the lever or by a trial-and-error process. The deviation of its body from the ideal beam of theoretical mechanics does not even allow this law to be applied directly to the ratio of its arms, contrary to what is often suggested in the literature. This also becomes clear from the fact that the zero point of the scale of a Roman steelyard does not, as a rule, coincide with one of its fulcrums and is often not even covered by the scale. The law of the lever can only be applied to a real steelyard in the form of a more sophisticated conclusion which can be derived from it, the conclusion that, in spite of its uneven and heavy beam, weight differences of the load remain proportional to differences in the positions of the counterpoise.³³

Another indication of the complexity of a real steelyard is the fact that practically all Roman steelyards preserved in Pompeii have more than one fulcrum and carry, accordingly, more than one scale. How could the builder of such a steelyard make sure that the weight-ranges of its two or even three scales smoothly joined each other? How could he select an appropriate counterpoise for a given balance with several scales? How could he estimate the range of weights that can be measured by a balance with given fulcrums? The Pompeian artisan could, in any case,

³² Generally, it is simply mentioned that the construction of the balance with unequal arms is based on the law of the lever, but not explicitly discussed how the scales were actually constructed, see e.g. Franken 1994, p. 13f. Some authors believe that the scale is not constructed theoretically but rather produced by gauging the balance empirically. Paret 1939, p. 77, for instance, gives a precise description of the way how the scales must have been produced, however, without any reference to sources which could provide evidence for the use of the alleged procedure. The complicated problems of designing the dimensions of a steelyard type balance with three consecutive scales harmonized with each other which cover a given range of weights are rarely mentioned in the literature. Jenemann 1989, p. 328, footnote 45, gives an algebraic formula for the calculation of the dimensions of such scales but does not propose any answer to the question of how the Romans might have constructed such balances without modern mathematics.

³³ If *G* denotes the weight of the movable counterpoise, *L* the distance between the suspension of the balance and the suspension of the load, and G_n and L_n the weight to be determined and the corresponding position of the counterpoise it follows from the law of the lever that the balance is in equilibrium if $G \cdot L_n = G_n \cdot L$, or $G_n/L_n = G/L$, for any constellation G_n and L_n . Subtracting the equations for two weights G_1 and G_2 from each other leads to the equation $(G_1 - G_2) L = G(L_1 - L_2)$, or $(G_1 - G_2)/(L_1 - L_2) = G/L$.

hardly tinker with the geometry of a balance in order to reach an optimal solution for this problem since the Roman balances consist of a beam cast in metal on which the positions of the fulcrums are fixed.

It is reasonable to assume that the Roman artisans followed certain practitioners' rules for dealing with these difficulties when designing the construction of a steelyard. It seems hardly possible, however, to reconstruct such rules from the ordinary archeological record, usually consisting of isolated samples from different places and times. A reliable reconstruction also appears difficult in the light of the usually neglected variance even of basic parameters of the artifacts such as the weights involved, as has been illustrated above for durable basalt weights from Pompeii.

It is only the unique archeological record due to the catastrophe of Pompeii that makes it conceivable to directly identify traces of practitioners' rules by analyzing characteristics of the surviving artifacts. We will briefly give two examples.

Among the Pompeian balances covered by our survey, there were 15 specimens of the steelyard type preserved well enough to study their geometry. Unfortunately some of the best preserved were not accessible to us because they were and are now again in an ongoing exhibition. All



Figure 5. RATIOS OF THE SCALE UNITS OF STEELYARDS WITH TWO SCALES

these steelyards have two scales. From an empirical point of view, there is no reason to stick to specific ratios between the length-units of these two scales. Plotting this ratio against the fifteen steelyards arranged according to the size of this ratio, one recognizes, however, a preference for certain ratios indicated by steps in the curve (see figure 5). This shows that there were rules to ensure that the two scales of a steelyard smoothly join each other, although further investigations are required in order to reconstruct the concrete rules that might have been applied.

Our second example again concerns the design of the scales. From the end of the 19th century onwards it is a stereotype in the literature on steelyards³⁴ that the weight of the counterpoises was empirically adapted to the difficult geometry of the steelyard and therefore did not represent a standard weight.



Figure 6. WEIGHTS OF THE COUNTERPOISES OF PRESERVED STEELYARDS

³⁴ See e.g. Franken 1994, p. 15; Paret 1939, p. 85.

Plotting the weights of the counterpoises of twelve of the fifteen steelyards (see figure 6), excluding those which, for different reasons, do not allow a precise value³⁵ to be determined in a similar way as we did in the case of the ratios, again shows steps which coincide with integer multiples of the libra or its half. The last section will argue that this is precisely what one has to expect according to practitioner rules transmitted by later Arabic literature.

Those rules have their basis in a relation between the geometry and the weight of the counterpoise which follows from the law of the lever although this consequence is not obvious and therefore is mostly overlooked in the scientific literature on balances.³⁶ The ratio between the distance of a fulcrum and the load suspension of the steelyard to the length of the corresponding scale unit determines the weight of the counterpoise. This makes it possible to determine the weight of the counterpoise belonging to a scale of a given steelyard if its geometry is known independent of whether the counterpoise is preserved or lost.³⁷

Calculated deviation								
2nd scale	1st scale	Deviation	Weight	Inventory number	Size			
30.6%	10.1%	6.5%	172 g	SAP 12934/60	0.5 libra			
destroyed	destroyed	15.2%	186 g	SAP 66/29.906				
destroyed	destroyed	-2.2%	316 g	SAP 1200	1 libra			
-7.6%	5.6%	1.5%	328 g	SAP 13448				
14.3%	18.3%	13.0%	365 g	MAN 74030				
10.4%	16.0%	15.8%	374 g	SAP 10280				
2.3%	destroyed	-5.3%	459 g	SAP 6521	1.5 libra			
-5.1%	-31.4%	-0.7%	481 g	MAN 744				
9.0%	5.3%	2.0%	494 g	SAP 13417				
-6.3%	19.1%	-1.5%	636 g	MAN 74051	2 libra			
-5.1%	1.2%	5.1%	679 g	SAP 11093				
5.7%	4.2%	3.2%	1000 g	SAP 4037	3 libra			

 Table 2: COUNTERPOISES OF STEELYARDS

Table 2 shows for the twelve selected steelyards, ranked according to the probable intended weights of their counterpoises as we have reconstructed them – the actual weight of the counterpoise in grams, the deviation from the reference value for the size of the libra and the devia-

³⁵ Inventory numbers of the excluded steelyards: MAN 74069 (badly produced steelyard with very imprecise scales), SAP 11231 (bad condition, scales not readable, heavily corroded), SAP 202000 (incomplete, scales have not been worked out, only three marks are at the beginning of the long scale, counterpoise missing).

 $^{^{36}}$ For an exception see Visy 1994.

³⁷ This is an immediate consequence of the generalized law of the lever mentioned above. If $(L_1 - L_2)$ is interpreted as the distance between two marks representing a weight difference $(G_1 - G_2)$ of one weight unit, and if the distance L between the suspension of the balance and the suspension of the load is known, the weight G of the counterpoise can be calculated from $(G_1 - G_2)/(L_1 - L_2) = G/L$.

tions calculated on the basis of the geometry of the steelyard for the two scales. The table first makes it evident that the counterpoises which are kept together with the beams in fact belong together. What is more interesting, the table reveals in some cases surprising differences between the qualities of the two scales of one and the same steelyard which can only be explained as errors in applying practitioners'rules. If the scales of a steelyard are correct, as is the case of the last row, the three deviations have to be the same. Usually, however, there are larger or smaller differences documenting the enormous difficulties an artisan had to confront when he was trying to design the geometry of a steelyard given the numerous constraints.

The remainder of this section is devoted to a brief description of some individual examples. That an ancient artisan was able to construct a steelyard that is not only precise but that also satisfies all other requirements of its functionality is demonstrated by a balance from the collection kept in Pompeii.³⁸ This steelyard displays two scales which perfectly join each other, one running from 1 to 6, the other from 6 to 25 librae, thus covering an impressive range of weights. The scales are not only carefully designed with Roman numerals for integer values of the libra and notches for the intermediate values, but also, as is shown by our measurements, astonishingly uniform and precise.

That beauty and functionality were not simply two sides of the same coin is illustrated by another steelyard, kept in the Museum in Naples.³⁹ This balance looks almost as perfect as the one just discussed and also displays scales which neatly fit to each other, one running from 1 to 5, the other from 5 to 20 librae. But it nevertheless suffers from a major flaw in its construction. For its first scale the ratio of the distance between fulcrum and load suspension to the unit of scale is close to 1, while the weight of the counterpoise is close to 1.5, which agrees with the corresponding ratio for the second scale. In other words, the values measured by the first scale of this balance are too small by a factor of 1.5, the second scale being approximately correct. While this balance would hardly have made a Pompeian merchant happy, it is precisely its error that provides us with a clue to the way such steelyards were produced, since it seems unlikely that such an error could be the result of a completely empirical procedure to construct the scales. Obviously there must have been a rule applied establishing a connection between the weight of the counterpoise and the geometry of the steelyard.

There are also balances whose appearance immediately suggests that they must have been produced by a dilettante or a beginner. This is true, in particular, for another steelyard kept in the Museum in Naples.⁴⁰ The weight of its counterpoise and the two ratios of the distance of ful-

³⁸ Inventory number SAP 13448.

 ³⁹ Inventory number MAN 74.4 (meaning of the dots unclear).
 ⁴⁰ Inventory number MAN 74069. This steelyard belongs to those that were not included in figure 6.

crum and load to the unit of the scale deviate considerably from each other in a range between 1.7 and 2.3. Moreover, the scales show a very low quality. Both scales only carry designations by numbers, while no notches are visible on the beam. Having a rather uniform first scale, the distances between the marks of the second scale vary considerably. Thus, the balance must have been very imprecise.

In steelyards which are produced by modern artisans⁴¹ such errors no longer occur. In particular, we could ascertain by our measurements of these modern balances that the ratios of the distance of fulcrum and load to the unit of scale practically always coincide with the weight of their counterpoises. Two millennia of artisanal tradition have evidently eliminated the variation between individually produced artifacts generating such errors.

4. THE FURTHER IMPACT OF PRACTITIONERS' KNOWLEDGE

In summary, our analysis points to what one may call the "spin-off character" of the law of the lever with regard to the technological development of balances with unequal arms. In fact, knowledge of the law of the lever is neither required nor sufficient to construct balances with unequal arms. But such knowledge could well be gained as a consequence of their construction. As was argued above, the first formulations of the law of the lever occur in theoretical texts which reflect the already available practical knowledge of such balances. In subsequent texts, the law of the lever then became the core principle of the emerging theory of mechanics.

This account raises the question of whether this further development then became independent of the knowledge of the practitioners or whether practical knowledge continued to have aftereffects on the science of mechanics. Is the realization of the sophisticated technology of balance-making that one can observe in Pompei merely to be considered as a technological refinement of once-and-for-ever established mechanical principles? Or does the practical knowledge involved in this technology continue to represent a challenge for theoretical mechanics?

⁴¹ This has been checked by investigating steelyard type balances which are still produced nowadays in a small manufacture in Italy (Fisciano, near Naples) and others produced by artisans in China (Beijing and Changsha). The balances were analyzed in the same way as the ancient ones, but showed, in contrast to them, no recognizable errors.

Ttol na füh E. 2. 4 5 540 0 E 26-1101.2. 2. Drin 1015 2 214 grand & on fear word' ble fit of tota la". db. fenful pride dy Actor - pana & pond. He Good 5 Il .Ac diula E ion Spin d. a. prid. 2. 2 6 the orgon can fare ab casi a bag. how you advord bb. ut present proved for ב פי בה שה שמיה קצי קחי הישה אש קצי שאוקה natio ab will ab Touphi . co. un . og. oun y . ag. Da ge oupluids Erts provab. tite by exceptial if if of anoth ag fent vis tol & exceltab. Tot as get fun eg inag eb & medutas bb & af we Epunderg. 2. aspond . 26 82 your liab. ast " and

Figure 7. Theorem on the material beam on folio 15 verso of a 15th century manuscript of the "Liber de canonio" (Foligno, Biblioteca Jacobilli, Ms. II 27)

4.1. THE MATERIAL NATURE OF THE BEAM

An answer to these questions evidently requires a thorough study of the long-range development of mechanical thinking and its relation to practical experience. But just a glimpse at subsequent milestones in the tradition of mechanical knowledge suffices to make clear that theoretical aftereffects of practical knowledge indeed existed. Our analysis of Pompeian steelyards has shown that one of the problems practitioners had to confront when constructing such a balance was the materiality of its beam whose weight had to be taken into account when gauging the scale. This problem is in fact, time and again, addressed in texts of theoretical mechanics, from the writings of Heron of Alexandria to the dozens of treatises left by Arabic scholars.⁴²

The "Liber de Canonio,"⁴³ for instance, a text of Hellenistic origin, is entirely dedicated to a material beam that is divided, as is the case with the beam of a steelyard or a Bismar, into two unequal parts (see figure 7). It provides a theoretical solution to a problem that must have also presented itself to the practitioners, that of compensating for the inequality in weight of the two sides of the beam by suspending an additional weight at the shorter end.



Figure 8. MANUSCRIPT PAGE OF BOOK II OF AL-KHAZINI'S "BOOK OF THE BALANCE OF WISDOM," WRITTEN ABOUT 1120 A.D., WHICH CONTAINS A DETAILED DESCRIPTION OF THE PROCEDURE TO CONSTRUCT THE SCALES OF A STEELYARD (ST. PETERSBURG, MS. KHANIKOFF, FOLIO PAGE 30 VERSO)

⁴² E.g. Thabit Ibn Qurra, al-Ahwazi, al-Isfizari, Ilya al-Matran, al-Khazini.

⁴³ Moody and Clagett 1960, pp. 64-75.

4.2. PRACTITIONERS' KNOWLEDGE IN THE ARABIC TRADITION AND BEYOND

In contrast to the extant literature from Greek and Roman antiquity, the Arabic tradition of mechanics explicitly deals with the construction of balances, thus revealing even more clearly points of contact between practical and theoretical knowledge. Complex balances are at the center of the work of the Arabic scholar al-Khazini from the beginning of the twelfth century.⁴⁴ The essential goal of his work is the perfection of a universal weighing instrument. In order to explain the construction and the function of this so-called "balance of wisdom," he assembles all practical and theoretical knowledge on balances and weights available to him.

As an intermediate step on his way to the balance of wisdom, al-Khazini also deals with the construction of the steelyard (see figure 8). His text offers amazing insights into practitioners'solutions of some of the problems raised earlier concerning the design of balances with unequal arms.

He shows, for instance, how a practitioner can easily determine, for a beam of given length and with given fulcrum and for a desired range of weights, the weight of the counterpoise:⁴⁵

"Then we take a compass and open it as far as we want to and we start dividing the beam from the position of the hook towards the point of the suspension. [...] The parts between the position of the hook and the point of suspension are rational to the opening of the compass. [...] Then one examines how many parts there are between the position of the hook and the point of suspension, and one takes a counterpoise whose weight is equal to the number of these parts measured in *mann*."

If, according to the practitioner's rule reported by al-Khazini, the scale unit is chosen such that the distance between load suspension and fulcrum corresponds to an integer number or fraction of these units, then the weight of the counterpoise must be equal to that same number of units measured in the standard of weight. This rule requires the use of a counterpoise whose weight is an integer multiple or fraction of the unit of weight. Al-Khazini's text has thus preserved a practitioner's rule that might provide an explanation for our surprising finding from Pompeii of counterpoises with weights of nearly integer or simple fractional values.

Even with al-Khazini's masterful work the potential of the balance as a challenge for theoretical mechanics was not yet exhausted. For the next 250 years mechanics essentially developed as a science of weights and of weighing, as it was in fact called both in the Arabic orient and in the

⁴⁴ See al-Khazini 1941, discussed in Knorr 1982.

⁴⁵ Following the designation of the chapters provided by Knorr 1982, pp. 206-212, the rule is contained in Book II, Part II, Chap. 3 of al-Khazini 1941, English translation by Mohammed Abattouy.

Latin occident. Only when the new technology of the Renaissance appeared within the intellectual horizon of mechanics, a profound transformation was initiated that eventually led to the classical mechanics of Galileo and Newton.

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