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Contents

1 Introduction

The history of the measurements of Earth's circumference in Antiquity is well-studied. Especially the cases of Eratosthenes, Poseidonios and Ptolemy have drawn much attention.[1] On the other hand, the relationship of the different methods and procedures employed have attracted much less interest. Of course, all these methods are sound from an astronomical point of view but have certain advantages and disadvantages from a more practical standpoint. Thus, historians of sciences used to think that the different results transmitted from antiquity, i.e. 250,000 or 252,000 stades for Eratosthenes, 240,000 or 180,000 stades for Poseidonios, 180,000 stades for Ptolemy are due either to certain imperfections in the astronomical observations or to some flawed presuppositions like the placement of Syene on the tropic of Cancer in Eratosthenes's measurement.[2] Also, different lengths for the Greek stade are often cited as an

[1] On Ptolemy's measurement see now Geus/Tupikova 2013.
[2] See, e.g., Geus 2002: 232–235.

explanation for the quite different results.[3] What we try to show in the following is that a) the length of the stade cannot explain the various results of Eratosthenes, Poseidonios and Ptolemy, b) that it was mostly Ptolemy's wrong presupposition of 500 stades (instead of 700 like Eratosthenes) to 1 degree that produced much confusion in the history of ancient astronomy and geography and c) that our correction of Ptolemy's presupposition show some remarkable results, not the least a new explanation for the distorted world maps of Ptolemy.

One of the most surprising features of Ptolemy's world map is its excessive distortion in the east-west-direction. The whole oikoumene from the Fortunate Islands (the Canaries) in the West till the metropolis of the Sines, Kattigara or Sera Metropolis (Xi'an in China?) in the Far East is equivalent to 180 degrees, surely more than third too high! A convincing explanation is still missing. Neither the assertion that Ptolemy subscribed to an aprioristic view that the oikoumene measures exactly half the circumference of the earth nor the fact that ancient eclipses were found with an intervall of 12 hours between both extremities (Ptol. synt. 2, 1, p. 88 Heiberg) of the oikoumene (see, e.g., Stückelberger/Mittenhuber 2009: 262) could have induced Ptolemy to draw such a distorted world map: We hear nothing about such an aprioristic belief in ancient sources – in fact, ancient geographers before Ptolemy have made vastly different calculations and guesses.[4] And as Ptolemy himself made clear in this introduction (*Geogr.* 1,4,1), he had only a few, if any, reliable observations of eclipses at his disposal and needed to make use of terrestrial measurements recorded in travelogues and itineraries. Thus, it seems obvious to attribute the east-west-distortion of Ptolemy's map to such inaccurate terrestrial measurements. In fact, Ptolemy's treatment of received measurements in his introduction seems quite sketchy and arbitrarily. Some errors of geographical localization in the *Geography* of Claudius Ptolemy are due to his method of calculating the routes:[5]

- for unknown routes the correction for uncharted stops and delays was considered as just 1/3 of the whole route;
- for voyages and expeditions along the coastlines or streets the deviations from the linear distance - the correction coefficient - were simply considered as a third;
- a route between two localities lying approximately on the same parallel was considered to proceed along this parallel (small circle) rather than along a big circle (geodetical line, the shortest route);
- the zero-point for calculation of the longitudes was put on the badly localized *Insulae Fortunatae* instead of some well-known city (like Alexandria in *Almagest*).

At first sight, it is not obvious at all why the physical size of the Earth should play any role in determining the geographical position of localities, such as latitude and longitude, expressed in degrees or even in dimensionless radiant measure. But is should be underlined here that it was not a data base in degrees that Ptolemy had at his disposal but the distances expressed in stades, dayruns and other units in use which he had to recalculate in angle measure to fit the world map under construction. In such recalculations, the adopted size of the Earth is of primary importance and the question whether Ptolemy used the same stade as Eratosthenes has fascinated the scholars since the rediscovery of Ptolemy's major work. Without standardisation of the metrical units in antiquity, no reliable answer can be found and the question of the resulting circumference of the Earth by Eratosthenes (252,000 stades) or by Ptolemy (180,000 stades) has been a kind of educated guesswork. Some estimations and the history of a subject are given in Appendix A. The impossibility to ascribe to Eratosthenes' value a satisfactory contemporary measure was the main reason for the al-Ma'mun's famous geodetical expedition (c. 830 AD) which produced a remarkable precise result for the circumference of the Earth.

Since all the confusing data produced very different (and far from agreed upon) results , we decided to suggest here an approach to tackle the problem in a new way. Instead of speculating about the modern

[3] For this see our survey of previous literature in Appendix A.

[4] Ptolemy's predecessor, Marinos of Tyre measured the oikoumene as being 225° in length.

[5] For discussion of some mathematical details one can see, e.g., a very commendable *Introduction* in J. L. Berggren and A. Jones "Ptolemy's *Geography*. An annotated Translation of the Theoretical Chapters", Princeton Univ. Press, 2000.

metrical value of a stadium used by ancient scholars, we will try to recalculate the geographical positions given by Ptolemy in his *Geography* in assuming that his definition of the stade used in the calculation of the geographical positions coincides with the definition of the stade used by Eratosthenes in his estimation of the circumference of the Earth. The comparison of the recalculated Ptolemaic coordinates for a (possible) "bigger" Earth with the modern values can confirm or reject our assumption about the equality of both Ptolemaic and Eratosthenes' stades and additionally throw light at the precision of early geographical mapping.

First of all, let us mention here the trivial fact that the simple "blowing up" of the sphere does not change the geographical coordinates (see Fig. 1). To transform the spherical coordinates of localities

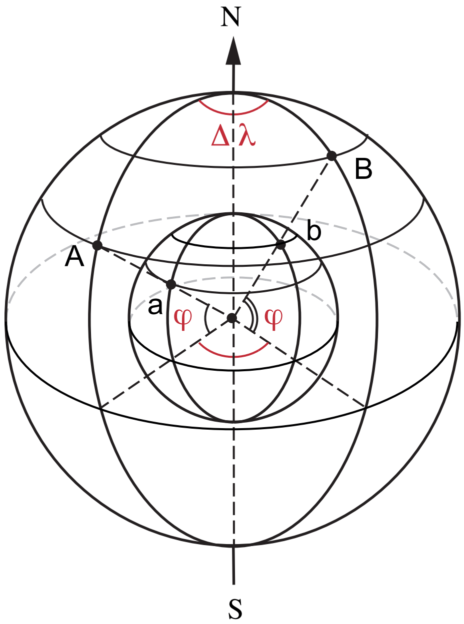

Figure 1: "Blowing up" of the sphere. The coordinates of the points a and b on the small sphere are given through φ_a, φ_b; the longitude difference is $\Delta\lambda_{ab}$. The projections of these points on the big sphere A and B have the same equatorial coordinates φ_A, φ_B and the same longitudinal difference $\Delta\lambda_{ab}$.

from a sphere with a radius r to a sphere with a radius R one needs, in fact, some formulae of spherical trigonometry. Although in all available textbooks on the subject the formulae are given for a sphere with a standard radius 1 only, the generalization for the case of a sphere with a non-unity radius and the transformation between the spheres with different radii is easy to perform.

One can argue that neither Ptolemy nor Eratosthenes knew or used such formulae; its is even proven that for the local mapping Ptolemy used simple plane triangles. Our answer to this possible objection is that first, we do not aim to improve the calculation technique used by Ptolemy in his geodetical exercises; our aim is to show how the set of his coordinates would look like if he had adopted as a scaling factor for his initial data in stades not the 500 but 700 stades per 1°. Second, the coordinates given by Ptolemy are already influenced by different kinds of inexactness and we do not want to corrupt them once more in the process of planar recalculations. Third, to find an approximative planar solution which would help to transform the planar triangles constructed for a sphere of one size to the planar triangles constructed for a sphere of another size is mathematically not trivial at all. Spherical trigonometry provides in this case a modern suitable and user-friendly device to recalculate the positions properly.

Whereas the many equivalent methods of determining the geographical latitudes were known in antiquity (through the relation of the longest day of the year to the shortest day or the relation of the length of the shadow of a gnomon to its length at midday during the summer solstices), the reliable determination of the geographical longitudes relative to a selected meridian (zero meridian) was not possible without some special kind of astronomical observations (e.g., simultaneously observed eclipses) and the help of precise time-keeping instruments. Such a measurement could not be performed before the introduction of the famous Harrison's chronometer H1 in 1735. Taking into account all these caveats, it is not surprising that previous scholars expected that the latitudinal coordinates given in *Geography* are in nearly all cases more reliable than his longitudinal data.

Nevertheless, what we try to show in the following is that the dominating error in the longitudes given in *Geography* can be, at least for the Mediterranean, reduced to a very small value, in many cases to the same level of precision as his latitudinal data.

2 Mathematical approach

To treat the problem with the methods of spherical trigonometry, one needs first to construct a spherical triangle. If the solution is aimed to improve the coordinates of one location relative to the other, the vertices of such a triangle are the localities themselves and the North (or South) pole on Earth's surface and the sides of this triangle are identical with the arcs of meridians going through both locations and the arc of a great circle connecting them.

We should also assume that the long routes were mostly lying on the great circles of the Earth's surface. Because the great circles describe the geodetical lines on the sphere, the shortest route between two points is always lying on the great circle – a fact unknown to Ptolemy. The deviations from these energetically preferable routes due to some landscape features or orientation problems can statistically balance each other out for big distances. The validity of such a presupposition (which, at first sight, seems quite hazardous) can be checked at the final stage of our recalculation in comparison with the real and recalculated positions of localities.

The first step of recalculation of the original positions is to restore Ptolemy's raw data, that is, the distances between different localities which he had at his disposal and – in some cases – the directions of the routes connecting these localities. Let us emphasize that without any information about the Earth's size, the exact geographical localization can not be unambiguously defined through the known latitudes and the known distance between two points. It can be shown that, depending on the Earth's size, different localities with different longitudes can be arrived via the same length of the route. To clarify this statement, let us suppose that some points A, B and a, b lie on the parallel circles with the same latitudes on the "big" and the "small" Earth (Fig. 2). Let us also assume that the arc of the great circle connecting localities A and B has a metrical value s (e.g., in stades) and that the longitudinal distance between A and B attains some value $\Delta\lambda_B$. On the "small" Earth the metrical value s will attain a bigger angular value and therefore, the location b lying on the same latitude (or polar distance) as B will have a *bigger* longitudinal distance $\Delta\lambda_b$ from the location a as $\Delta\lambda_B$. Therefore, some additional information (e.g., the course angle or the size of the Earth) is necessary to find out the longitude in question, that is, to determine the geographical position of the locality B relative to A. On the other hand, with the known course angle but without knowing the length of a route, the problem has mathematically two possible solutions – a fact which was probably unknown to Ptolemy.

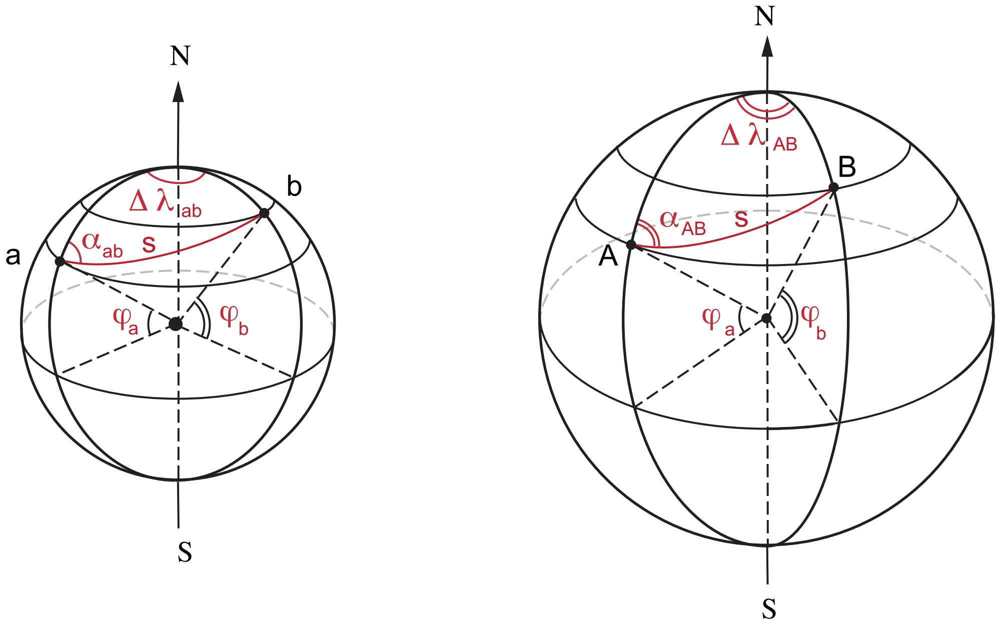

Figure 2: Two localities with known latitudes and known mutual distance. Points a and b on the “small” Earth lie on the same latitudes as the points A and B on the “big” Earth respectively. With the same known distance s between the localities, the longitudinal distance $\Delta\lambda_{ab}$ is different from the longitudinal distance $\Delta\lambda_{AB}$. The course angle α_{ab} connecting the locations a and b is also different from the course angle α_{AB} connecting the locations A and B. To find out the geographical positions *unambiguously*, one needs additional information about the course angle or the real size of the Earth.

To recalculate the geographical positions of the localities on the “real” Earth we should consider different methods, depending on the information available to Ptolemy.

Case 1

The first procedure can be applied to the places with correctly determined (for example, by astronomical observations) geographical latitudes. For these localities only the longitudes should be corrected. Let us consider two places: a (the latitude φ_a) and b (the latitude φ_b) with the longitudinal difference $\Delta\lambda_{ab}$ as given by Ptolemy for the “small” Earth (see Fig. 2, left). The transformation to the positions A and B on the “big” Earth (Fig. 2, right) will not change the latitudes of the localities $\varphi_a = \varphi_A$, $\varphi_b = \varphi_B$. From the formula for the circumference of a circle, the radius of the “small” Earth (i.e. Ptolemy’s Earth), r, can be found as

$$r = 180,000/(2\pi)$$

and the radius of the “big” Earth (i.e. Eratosthenes’ Earth), R, as

$$R = 252,000/(2\pi).$$

To recalculate the longitudinal difference, we need, in fact, Ptolemy’s “raw data”, the length of a route from a to b in stades. It can be found first in radian measure from the cosine-theorem

$$\cos s = \cos(\pi/2 - \varphi_a)\cos(\pi/2 - \varphi_b) +$$
$$+ \sin(\pi/2 - \varphi_a)\sin(\pi/2 - \varphi_b)\cos\Delta\lambda_{ab}$$

and then recalculated in stades as

$$s_{st} = s * r$$

(that is, in fact, the value for the length of a route available for Ptolemy). This value can now be recalculated in radian measure for the "big" Earth as

$$S = s_{st}/R.$$

Now, the "true" longitudinal difference $\Delta\lambda_{AB}$, that is, the longitude which Ptolemy would have found out if he had used Eratosthenes' estimation of the size of the Earth can be calculated in radian measure according to

$$\cos\Delta\lambda_{AB} = \frac{\cos S - \cos(\pi/2 - \varphi_a)\cos(\pi/2 - \varphi_b)}{\sin(\pi/2 - \varphi_a)\sin(\pi/2 - \varphi_b)}$$

where $\varphi_a = \varphi_A$ and $\varphi_b = \varphi_B$. The value of the longitudinal difference in degrees is given through

$$\Delta\lambda^{\circ}_{AB} = \Delta\lambda_{AB} * 180^{\circ}/\pi.$$

This is why Ptolemy's world map seems to be elongated in the east-west direction. Among the few reliable data which were available to Ptolemy at his time, the rare latitudinal values of some prominent locations laid the groundwork for Ptolemys mapping, the staging of the whole construction. The terrestrial distances between the localities, transmitted for the most part by merchants or soldiers, were used in determining the longitudinal coordinates. Due to the erroneously adopted size of the Earth, Ptolemy should have consequently obtained a bigger longitudinal difference for every pair of locations with known latitudes and known distance between them (see Fig. 2 and Fig. 3, points B and b, respectively).

Case 2

The second procedure can be applied when recalculating the coordinates of the place c lying at a latitude (φ_c) which was unknown to Ptolemy. The geographical position of such a locality must have been calculated by Ptolemy only on the basis of the length of the route and the estimated angle of the route connecting this locality with with a starting point a with the known latitude φ_a. In this case, the geographical latitude of c as well as the difference in longitudes $\Delta\lambda_{ac}$ between two localities should be corrected.

To recalculate the coordinates on the "real" Earth's surface, one can proceed in the following way. First, we must restore the length of the route and the course angle available for Ptolemy. The arc of the route s in radian measure can be found exactly as in the first case from the cosine theorem

$$\cos s = \cos(\pi/2 - \varphi_a)\cos(\pi/2 - \varphi_c) + \\ + \sin(\pi/2 - \varphi_a)\sin(\pi/2 - \varphi_c)\cos\Delta\lambda_{ac}.$$

The length of the route in stades can be restored in the same way as

$$s_{st} = s * r$$

The course angle α can be found with the help of the sine theorem from

$$\sin\alpha = \frac{\sin\Delta\lambda_{ac}\sin(\pi/2 - \varphi_c)}{\sin s}.$$

Because this value of α was, in fact, measured on the real Earth's surface, we should keep this value for further calculations. With these restored data, s_{st} and α, Ptolemy would have obtained his erroneous values for the latitude of the second locality φ_c and the longitudinal difference $\Delta\lambda_{ac}$. To calculate the corrected values, we need the value of the route S in radian measure on the "real Earth"; it can be calculated once more, as

$$S = s_{st}/R.$$

Now, we can find the "true" latitude of the locality C through the cosine theorem

$$\cos(\pi/2 - \varphi_C) = \cos(\pi/2 - \varphi_a)\cos S + \sin(\pi/2 - \varphi_a)\sin S \cos\alpha$$

where $\varphi_a = \varphi_A$.

The longitudinal difference $\Delta\lambda_{AC}$ can be found with the help of sine-theorem in the radian measure as

$$\sin\Delta\lambda_{AC} = \frac{\sin\alpha \sin S}{\sin(\pi/2 - \varphi_C)}$$

and recalcuted in degrees through

$$\Delta\lambda^\circ_{AC} = \Delta\lambda_{AC} * 180^\circ/\pi.$$

Fig. 3 (points C and c, respectively) shows schematically how a map will be distorted in this case. The geographical coordinates on the "small" Earth show the overexpansion along the east-west as well as along the north-south direction.

Case 3

In some special cases, the positions of the locations slide in the coordinate grid only along the north-south direction.

Let us consider a pair of localities lying on the same meridian. The lists of such cities, *antikeimenoi poleis*, circulated in antiquity since the time of the pre-geographical mapping as a means of a rough orientation between major cardinal points like important cities, ports or landmarks.

Let A be a reference point lying at same latitude φ_A on the "big" Earth and D at some distance S from A measured along the meridian with the unknown latitude. If the distance S is expressed in stades and we accept the length of a degree on the Earth surface being 700 stades, the point D will lie $S/700$ degrees to the south of A (Fig. 3, right). On the "small" Earth, where $1^\circ = 500$ stades, the appropriate point d will lie $S/500$ degrees to the south of A (Fig. 3, left). In this way, the latitudinal difference between a and d will attain $7/5 = 1.4$ of latitudinal difference between A and D - that is, the point d will be placed more to the south in relation to its actual position. Accordingly, a point lying at the known distance to the north of A, will be moved towards north on the "small" Earth in comparison with its position on the "big" Earth.

If a locality lies not exactly on the meridian of A but closely to it, its position on the "small" Earth will move more to the south (or more to the north) relative to its actual position and will also show a small latitudinal displacement (points F and f in Fig. 3). This case can be treated with the formulae of Case 2.

Although only rarely, the cases of unexpected latitudinal displacement can be observed on Ptolemy's world map. One of these instances, the notorious displacement of Carthage, will be discussed later.

Case 4

The case of a locality lying on the meridian of the reference point with the known latitude *and* with the known distance was a much more challenging for Ptolemy. With the angular measure of this distance on the "small" Earth he was not able to reach the known latitude of such a locality. In this case, as a professional astronomer who puts more trust in the "meteoroscopial method", he just would have preferred to transmit the known latitude of a locality on his map (Fig. 3, points E and e, respectively) and dismiss the less reliable distance measure. Such cases can be easily detected on the Ptolemy's map; from a mathematical point of view, it is more complex to recalculate the position of localities in vicinity of such points with "transmitted" latitudes. Although the positioning of such localities matches very well their actual position, they are strictly speaking "not from this map" and the coordinates of the localities which were adjusted in a local map around such "alien" locations through their relative position to the *initial* reference point will be generally distorted.

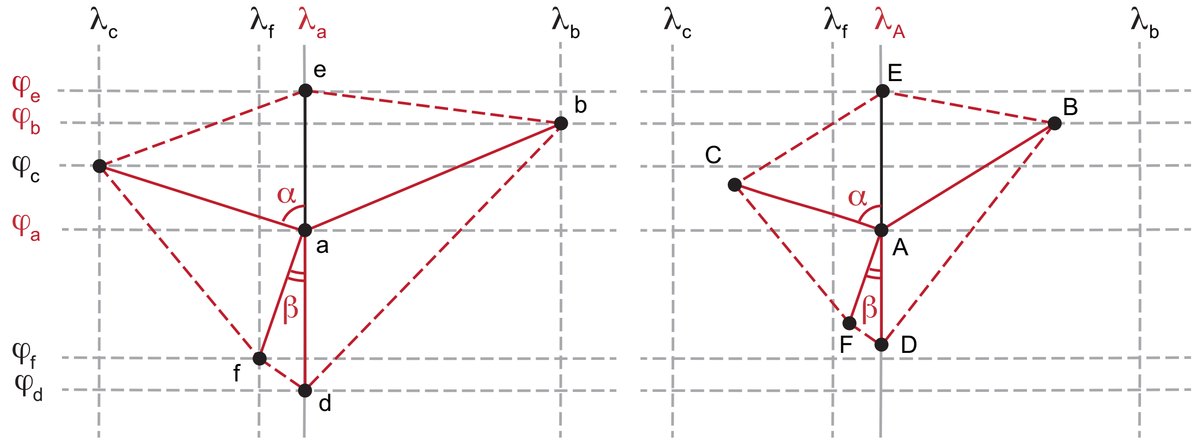

Figure 3: Schematical illustration of the possible cases of distortion in Ptolemy's world map. Left: "small" Earth. Right: "big" Earth. The point $a = A$ is the starting point of the mapping. The known quantities are marked with red color.

Case 5

For some values of the latitudinal difference between two points, the latitude of the second point could not be reached with the given length of the route on the "big" Earth (see Fig. 4). Our program[6] sorts out such cases and proposes an alternative reference point in this case.

[6]In our calculations we used the software system MAPLE 12; the program can be made available upon request.

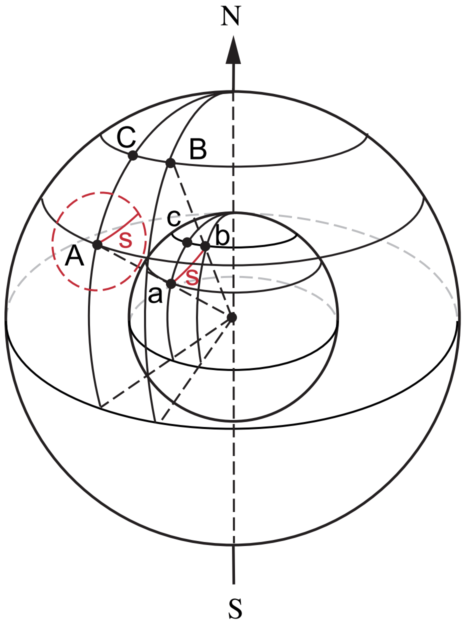

Figure 4: Two localities with known latitudes φ_a and φ_b lie at a distance s. Points A and B on the "big" Earth lie on the same latitudes as the points a and b on the "small" Earth respectively. For some values of the latitudinal difference between two points, the latitude of the point B can not be reached with the same length of the route s.

Of course, Ptolemy would also have faced a mathematical problem with adjusting geographical coordinates for the case of three localities with known latitudes and with the known routes between them. For some cases the routes do not fit together because of the adopted erroneous value for the circumference of the Earth.

3 The Problem of the Prime Meridian

Having corrected the positions in relation to any chosen reference point, we face now the problem of comparing the recalculated positions of the localities with their actual geographical location.

The modern geographical coordinate system is a spherical one with the Earth's equator as a reference plane. Latitudinal values (φ) are measured relative to the equator along the meridian of a location and range from $-90°$ at the south pole to $+90°$ at the north pole. Longitudinal values (λ) are measured relative to the prime meridian. Since the International Meridian Conference in 1884 the meridian of the Royal Observatory in Greenwich (UK) has been established as the zero-reference line, the Prime Meridian. The latitudes range from $-180°$ when traveling west to $+180°$ when traveling east from Greenwich.

The positions in Ptolemy's *Geography* are established practically in the same coordinate system. In fact, it was his treatise where for the first time the spherical equatorial coordinate system was subsequently applied for geographical mapping. Whereas the Ptolemaic latitudes can be considered as being equivalent to the modern values, his longitudinal values should be corrected for the position of his Prime Meridian in relation to the modern Greenwich meridian.

In the second book of his *Syntaxis mathematike*[7] Ptolemy announced to compile a list of important ancient cities, *Poleis diasemoi*, with their geographical positions: latitudes measured from the equator and longitudes measured from the prime meridian placed at Alexandria. In his later work, in the *Geography*, the positions of the localities were defined, in contrast to his announcement, relative to a prime meridian

[7] *Syntaxis mathematike*, 2,13, p. 188 Heiberg.

(*Geogr.* 1,11,1) going through the *Insulae Fortunatae*.[8] The position of Alexandria relative to the *Insulae Fortunatae* is given as $60°30'$.[9] How can the longitude of the modern Prime Meridian, the Greenwich meridian, be determined relative to the position of the Ptolemy's prime meridian? The modern longitude of Alexandria is about $29°55'$ E relative to the Greenwich meridian. Subtracting this value from Alexandria's longitude given by Ptolemy (see Fig. 5), we can obtain the longitude of Greenwich relative to Ptolemy's prime meridian:

$$60°30' - 29°55' = 30°35'.$$

It we try to recalculate the position of Ptolemy's prime meridian through the coordinates of Rome (Ptolemaic position $36°40'$, modern position $12°29$), we obtain

$$36°40' - 12°29' = 24°11'.$$

What we try to show here is that whatever identified location will be chosen, the position of the Greenwich meridian relative to the *Insulae Fortunatae* will always be different. The correct mathematical definition in the strange geometry of Ptolemy's distorted world would be something like "the position of the modern Prime Meridian relative to the Ptolemy's zero meridian is defined as the longitude of the location for which the calculated Greenwich's longitude attains zero value."

The problem does not lie in the poor determination of the positions in Ptolemy's time: it lies in Ptolemy's attempt to map the available distances onto a sphere of a wrong size. As a result, all the local regions of Ptolemy's maps are distorted relative to *every* starting point of his distances data. The maps are stretched along east-west lines for the localities with known latitudes and along all the other possible directions in other cases (see Fig. 3). This is why the identification of the position of the Greenwich meridian through the modern coordinates of identified localities is always delicate – it slides along the modern coordinate system. As a consequence, it can not be linked with the Ptolemaic coordinates *globally.* In our view, it makes no sense to speak of the position of the Greenwich meridian relative to Ptolemy's zero meridian without mentioning the chosen reference point.[10]

In our recalculations we proceed in the following way. First, the position of the Greenwich meridian, λ_G^P will be defined relative to a reference point (A) with the known Greenwich longitude λ_A^G as

$$\lambda_G^P = \lambda_A^P - \lambda_A^G$$

where λ_A^P is the longitude of A as given by Ptolemy.

Second, the Greenwich longitude of another locality B whose position should be improved relative to a reference point A is found through the following formula (see Fig. 5):

$$\lambda_B^G = \lambda_A^G - \Delta\lambda = \lambda_B^P - \lambda_G^P$$

where $\Delta\lambda$ can be found as the difference in the Ptolemaic longitudes of both localities. The recalculated coordinates of B are now linked with the position of the Greenwich meridian found through the reference point.

[8] In *Geogr.* 8,3–28, the positions of *Poleis diasemoi* are given relative to meridian of Alexandria in hours and the latitudes in durations of the longest days of the year. According to A. Stückelberger (in Stückelberger / Mittenhuber 2009: 139), these values were recalculated from the catalogue of the locations. The position of Alexandria relative to the *Insulae Fortunatae* is given in *Geogr.* 8,15,10 as 4 hours = 60°.

[9] *Geogr.* 4,5,9.

[10] It makes even less sense to define the position of Ptolemy's Prime meridian relative to the place where at a later time the Greenwich Observatory should be placed because Ptolemy's map of England is grossly distorted.

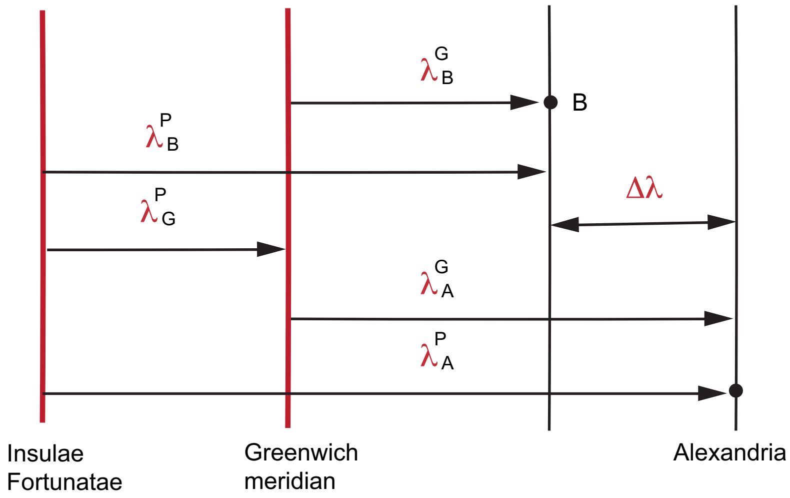

Figure 5: Determination of the position of the Greenwich meridian through the modern and the Ptolemaic positions of a locality with the example of Alexandria. The longitude of Alexandria relative to Greenwich meridian is marked with λ_A^G, the position relative to the *Insulae Fortunatae* as λ_A^P and the position of the Greenwich meridian relative to the *Insulae Fortunatae* as λ_G^P. This position will be different for the various reference points due to the distortion of Ptolemy' world map.

4 Recalculation of Ptolemy's coordinates

In order to make our results "more visible", only some important cities which played a major role as ancient ports and hubs are displayed on our recalculated map of the Mediterranean. In the following illustrations, the black circles with black numbers indicate the modern positions of locations, the red circles with red numbers the positions as given by Ptolemy in his *Geography* recalculated for the Greenwich longitude determined relative to a selected reference point (as discussed in the previous section). The numbers indicate the names of the locations which are given together with their original Ptolemaic and recalculated coordinates in Appendix B.

We have started first with the recalculation of Ptolemy's coordinates for the eastern part of Mediterranean.[11] It seems reasonable to suggest that the longitude of Alexandria, the hometown of Ptolemy, relative to the *Insulae Fortunatae* was best known to Ptolemy and served as his starting point. Thus, the "reduction to Greenwich" defined in relation to Alexandria, $30°35'$, is subtracted from Ptolemy's longitudes of the locations whose coordinates we try to correct. In such a way, these locations are linked with our modern coordinate system.

It is to be expected that, at this scale, the error in the determination of the Earth's size does not manifest itself so drastically as at the extremities of the *oikoumene*; on the other hand, the positions of the locations were better known and the results can be easily verified.

Some positions of prominent cities recalculated for the circumference of the Earth equalling to 252,000 stades through the spherical triangle with the vertices in Alexandria, North Pole and a locality are given in Fig. 6 and marked with yellow circles; the numerical results are given in Appendix B, Table 1.

The results can be allocated to four classes. The most striking result is the improvement of the positions in the western part of Sicily: the latitudes of the localities are approximately accurate and after our recalculation the longitudes show a very good match with the modern longitudes. To the second class one can attribute the Italic positions as well as Sardinia and Carthage. In general, one can see an obvious improvement of the positions which lie always east from the true positions, whereas the Ptolemaic coordinates lie always west and at a greater distance from them. What is more, the whole Italy shows

[11] We have taken the coordinates of the Ptolemaic localities from the new edition by Stückelberger/Graßhoff (2006).

a systematically reduced latitudes' value (confirmed by K. Guckelsberger, private communication) and a relatively big longitudinal displacement even after the recalculation to the "big" Earth's size. Three of localities displayed in Fig. 6, Olbia, Marsala/Lilybaion and Carthage, show a remarkable latitudinal error; the graphical pattern of their true, Ptolemaic and recalculated positions points to a common source of error and a possible link between their mapping by Ptolemy. To the third class belong Athens and Thessaloniki whose positions – the Ptolemaic as well as recalculated ones – are totally misplaced. This is especially eye-catching because Athens, e.g., can be reached directly via sea from Alexandria and its latitude was very well known. The localities of the fourth class – Rhodes, Samos and Smyrna – are in fact at their actual positions and can not be further improved (mathematically, the problem has no solution). As a working hypothesis, we adopted an explanation discussed in Case 4.

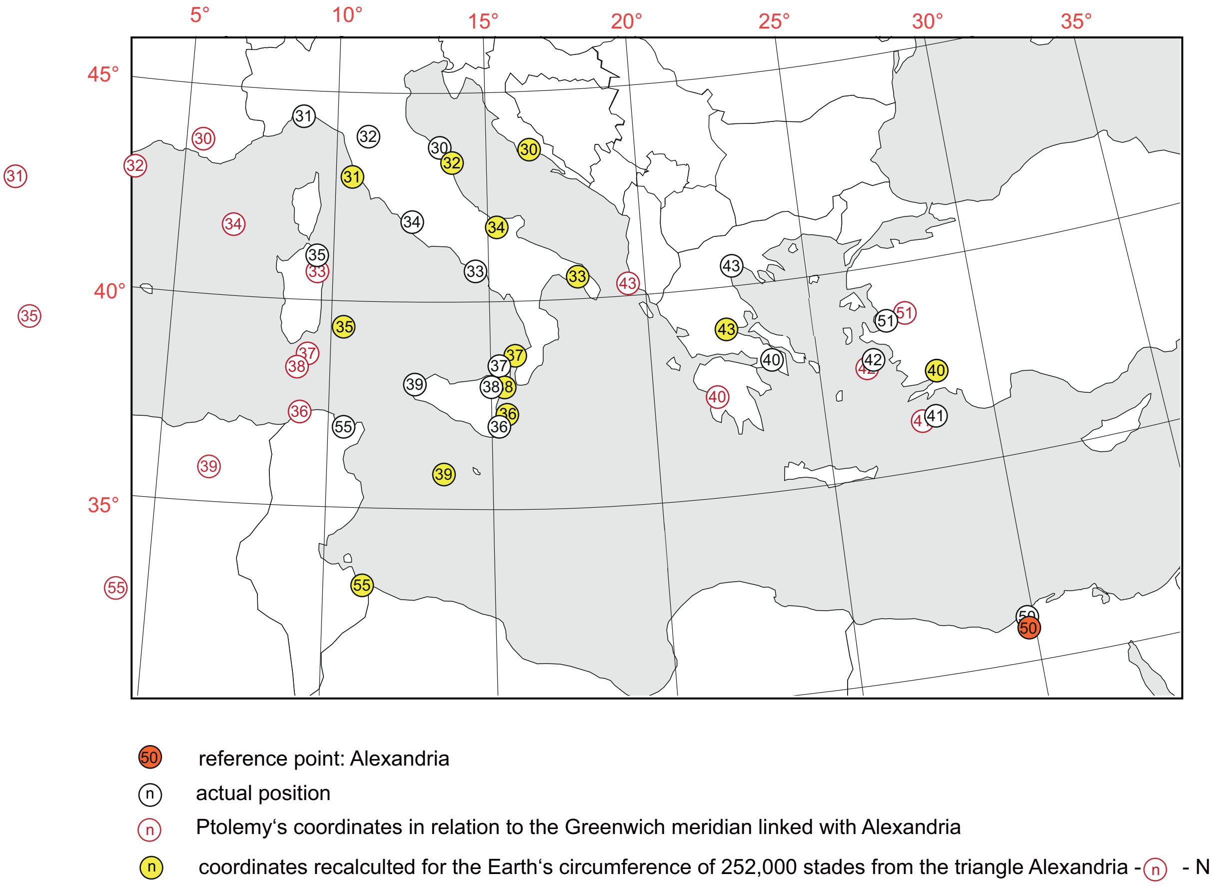

Figure 6: Recalculation of some prominent locations in the eastern Mediterranean for the circumference of the Earth equal to 252,000 stades. The reference point for this recalculation is Alexandria.

This special pattern of the distribution of the Ptolemaic and the corrected (for the "big" Earth) coordinates forced us to suggest that a significant part of the Ptolemy's map for this region was constructed on the basis of the local Italic distances only and corrupted due to the erroneous longitudinal alignment to Alexandria. In such thoroughly measured land as Italy, the distances relative to Rome would have been known rather accurately and with the recalculation of these distances for the bigger size of the Earth the coordinates could possibly be improved. In order to eliminate the expected longitudinal error of Rome relative to Alexandria, we have chosen for our map Rome's position (strictly speaking, the coordinates of the famous *Milliarium Aureum* at the Forum Romanum) as the reference point and recalculated the Ptolemaic coordinates first for the "Greenwich reduction" relative to Rome, 24°11′. Then, the coordinates were recalculated for the "big" Earth. The results are shown in Fig. 7.

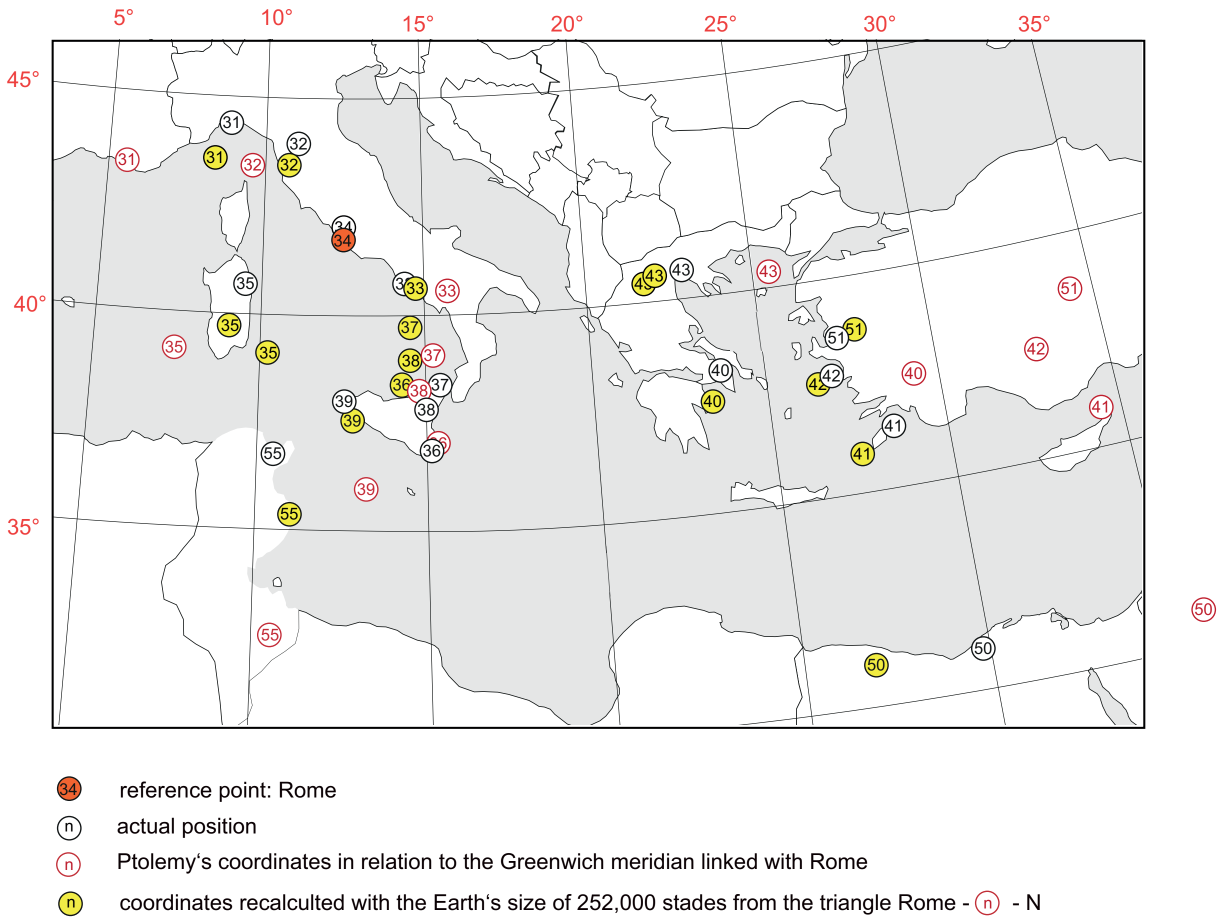

Figure 7: Recalculation of some prominent locations in the eastern Mediterranean for the circumference of the Earth equal to 252,000 stades. The reference point for this recalculation is Rome.

Our recalculations show a very different pattern, not the least a much better precision of the map. Most interestingly, the adjusted coordinates of Thessaloniki and Athens exhibit now a nearly perfect match with their real ones. This makes a very good case for claiming that Ptolemy has measured this part of the Mediterranean not from Alexandria but from Italy, most possibly Rome.

Nearly all results were obtained with the formulae of Case 1. In two instances, for Olbia and Thessaloniki, the formulae of Case 2 give a slightly better result; for these cases we show both adjusted positions. One cannot exclude for either city that the knowledge involved here was rather based on course angles than on distance, albeit this seems to be less likely from a navigational point of view (the ancients mariners used only a rough wind compass and were travelling along the coastlines)

The location of Marsala can not be recalculated from Rome with the formulae of Case 1 because of the problem discussed in Case 5. It can nevertheless be recalculated with the formulae of Case 2 and the coordinates match very well its real position. The case of Marsala as well as the recalculation of the position of Carthage (which shows after our adjustment only a latitudinal error of 1.8 instead of the notorious error of 4) strengthens our hypothesis of the underlying mathematics of the whole “distortion problem”: the coordinates can be improved in agreement with our prognosis discussed in Case 3. It explains also why Ptolemy has misplaced both localities so much to the south – it can be explained now as a result of the adoption of a smaller size of the Earth while mapping positions with unknown latitude not far from the meridian of the reference point (in this case, Rome).

The coordinates of the localities in western of Sicily can not be recalculated from Ptolemy’ coordinates with the formulae of Case 1 – we face here a case displayed in Fig. 4. The recalculation of these positions with the formulae of Case 2 shows a failure discussed in Case 3: the positions would move towards the north. In fact, the position of Syracuse at the western Sicily shows already a remarkably good latitude. We think that both the known value of latitude and the value of the longitude gained from the estimated relative position of this city to Rome were just aligned to the map (Case 4, "alien" point).[12] As a consequence, the coordinates of the western coast-line points which were adjusted in a local map around Syracuse are generally distorted.

The recalculated Ptolemaic coordinates of Rhodes, Samos and Smyrna can also very well been corrected for their longitudinal displacement. This fact, in our opinion, means that originally their coordinates came from Italian sources (otherwise they could not be improved after recalculation to the "big" Earth's size) and the position of Alexandria was inevitably linked with these localities with well known latitudes and mutual positions (Case 4). It is also easy to see from their recalculated positions that Alexandria was linked badly with Rhodes and Samos. Again, the problem of the erroneous size of the Earth pops up here. If Ptolemy had chosen to preserve the accurate distances to these islands, he should have placed Alexandria far more in the eastern direction as he did in the end. As a result, this part of Mediterranean is not coherent with the rest of the map. Alexandria as well as the localities linked with it, all have wrong longitudinal alignments and should therefore be considered as an own case

Let us recall, that we have not ascribed any modern metrical value for the stade. We have just recalculated spherical coordinates for a sphere with a circumference of 180,000 untis to a sphere with a circumference of 252,000 units and compared the results with the modern coordinate system while adjusting the meridian of a reference point with his modern position. With this error been corrected, Ptolemy's map achieves a spectacular level of precision.

According to the data base for the ancient distances collected by the second author[13] of the paper from different sources, the most cited reference points are in the this order: Rome, Carthago, Mons Calpe, Alexandria and Babylon. Because of that, we have also chosen Rome as a reference point to recalculate the Ptolemaic positions for Spain and France. The preliminary results are shown in Fig. 10.

Because at such huge distances relative to the reference point the erroneously adopted size of the Earth manifests itself at a bigger scale, the results of the correction for the Earth's size are also much more prominent (see Fig. 10). The distribution of recalculated coordinates shows different patterns in France and Italy (the numerical results are given in Appendix B, Table 3). The recalculated locations of Tolosa (Toulouse), Lugdunum Metropolis (Lyon), Burdigala (Bordeaux) and Massilia (Marseille) are of a remarkable precision – it is obvious that their positions were in fact defined in relation to Rome. The coordinates in Spain are also drastically improved but show another displacement's pattern: all the recalculated coordinates lie in the west of the actual positions. Because a small displacement in the same direction shows also the coordinates of Marseille, one can suggest that its location served as a starting point for Ptolemy's mapping of Spain. This aspect merits further investigation.

The best results in comparison to the modern coordinates were obtained in all cases with the formulae of Case 1. That means that for these localities the latitudes of the cities were well-known and the longitudes were determined by Ptolemy with the help of the estimated routes only.

[12]In such a case, recalculation of the relative distances was unnecessary. Therefore, the error induced by the adopted false Earth' s size was avoided.

[13]This data base on ancient measurements is a project carried out in the excellence cluster TOPOI (Berlin).

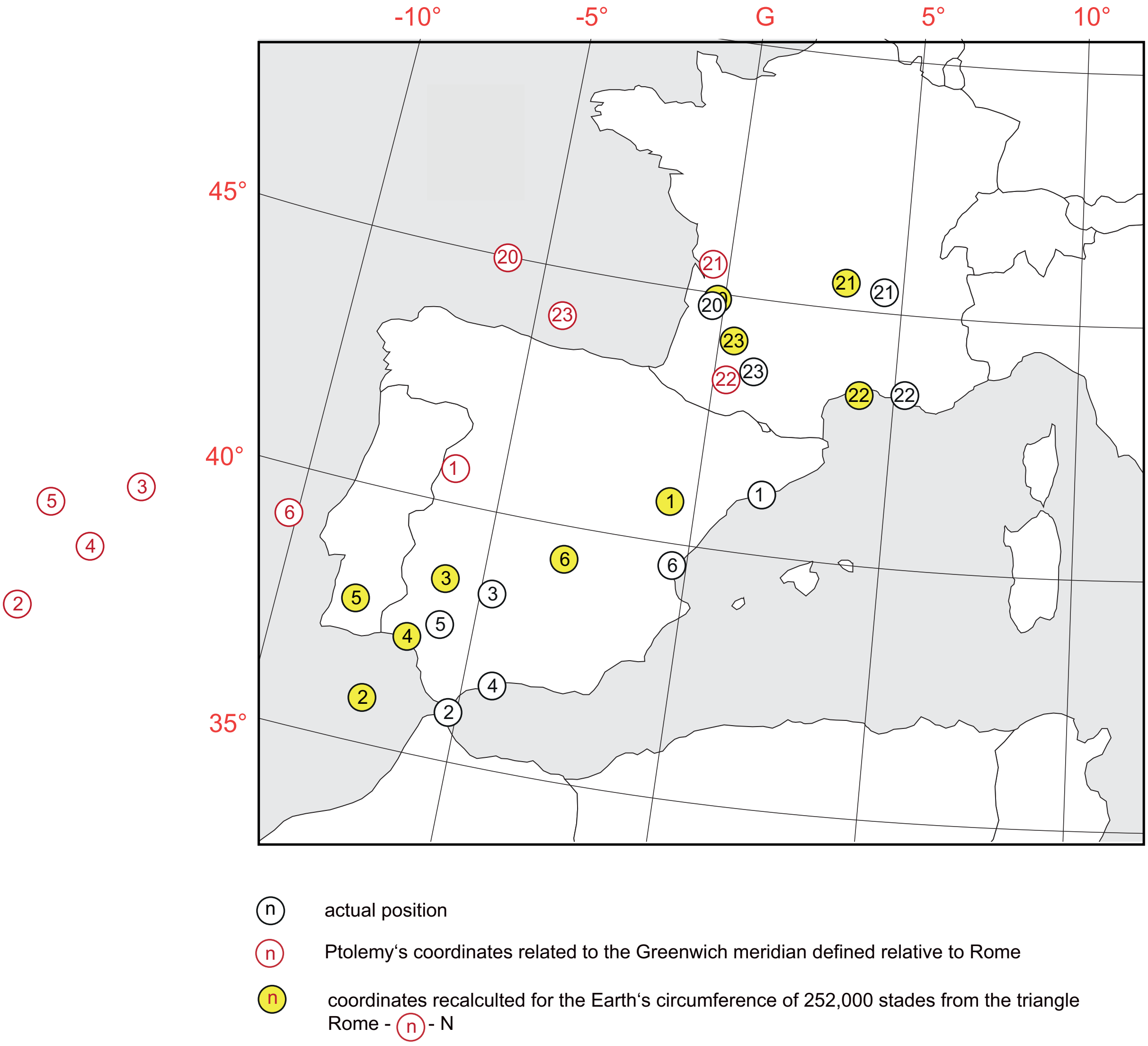

Figure 8: Recalculation of some prominent locations in the western Mediterranean for the circumference of the Earth equal to 252,000 stades. The reference point for this recalculation is Rome.

5 Russo's estimation of the metrical value of a stade

In the process of preparing this paper, we became aware of about L. Russo's online publication *Ptolemy's Longitudes and Eratosthenes' Measurement of the Earth's Circumference*. The main idea of the publication is to compare statistically the longitudes reported in Ptolemy's *Geography* with the actual positions of identified locations. With a sample of about 80 locations, the author has obtained an empirical formula

$$\lambda_{Ptolemy} = 1.428\lambda_{modern} + 17.05°$$

connecting Ptolemy's longitudes with their real counterparts. The term 17.05° was interpreted by the author as the longitude that Ptolemy would have assigned to Greenwich meridian (for our opinion to this topic see chapter 3). This statistically acchieved formula was applied by the author to estimate the value of a stade used by Eratosthenes under the assumption that both ancient shoolars used the same notion of stade. The author proceeded in the following way. Let $\Delta\lambda_{modern}$ be the difference in longitude between two arbitrary locations, $\Delta\lambda_{Ptolemy}$ is their longitudinal difference according to Ptolemy, d_m and d_s are the metrical values of an arc of equator between the meridians of these two localtion in meters and stades, respectively. Because at equator the length of 1° is approximately 111100 meters and because we know that Ptolemy attributed to 1° a value of 500 stades, we can get the value of a stade in meters as

$$stade = \frac{d_m}{d_s} = \frac{1111000}{500} \frac{\Delta\lambda_{modern}}{\Delta\lambda_{Ptolemy}}.$$

By replacing the ratio $\Delta\lambda_{modern}/\Delta\lambda_{Ptolemy}$ with its mean value given by the regression coefficient, 1/1.428, the author has obtained for the stade the value of 155.6 meters. Of course, this value is a purely statistical result and one can argue whether to find a metrical equivalent to a stade in the contemporary units one should better take into account the best known localities.

Although the author has not recalculated the coordinates given in *Geography*, we have noticed that his empirical statistical formula can be applied, in fact, to every Ptolemaic coordinate to recalculate the new longitude, that is, the longitude which Ptolemy would have obtained with the adopted circumference of the Earth of 252000 stades in form

$$\lambda_{recalculated} = (\lambda_{Ptolemy} - 17.05^\circ)/1.428$$

Being not an accurate recalculation, this formula (which we will call the "regression formula"), can be considered as an approximative polynomial of the first grade and it can provide for a chosen locality in some sense "an averaged value of the longitude referred to an averaged position of the Greenwich meridian". As a test for our own recalculations we have applied this idea to some selected locations and noticed a remarkably good match with our own recalculations and with the actual coordinates of the locations. The comparison with our recalculations are shown in Fig. 9 for Italy and Greece and in Fig. 10 for Spain and France. The good adjustment to an actual position given by the regression formula means that the error in the geographical coordinates of a location is near to a middle value, the significant deviation points to an erroneously linked position relative to a "mean value" of the prime meridian. The coordinates recalculated with the statistical formula are not flexible and can not be recalculated relative to a selected meridian. To show, what the choice of a reference meridian can mean for further studies, we have recalculated the positions given in Fig. 10 also for Marseille as a reference point. One can see that in this case the positions of cities in France are hardly improved which points towards their alignment with another source, almost certainly Rome. The recalculated positions in Spain in contrast became significantly improved relative to the positions given by the regression formula: an explanation would be that the Spanish positions were taken in Ptolemy's map through their known distances to the haven of Marseille and inherited Marseille's longitudinal error.

Although Russo's regression formula can not reveal important details given in our algorithm such as identification of the starting point for Ptolemy's distances base or localization of the "dead zone" due to the adopted erroneous value of the circumference of the Earth (Case 5) or improve the position of the locations as in Cases 2 and 3, it certainly can be used for approximative researches and for statistical investigations. The sample of Russo's locations must nevertheless be expanded to other regions. His statistical formula should not, in fact, be necessarily linear over the whole world map of Ptolemy.

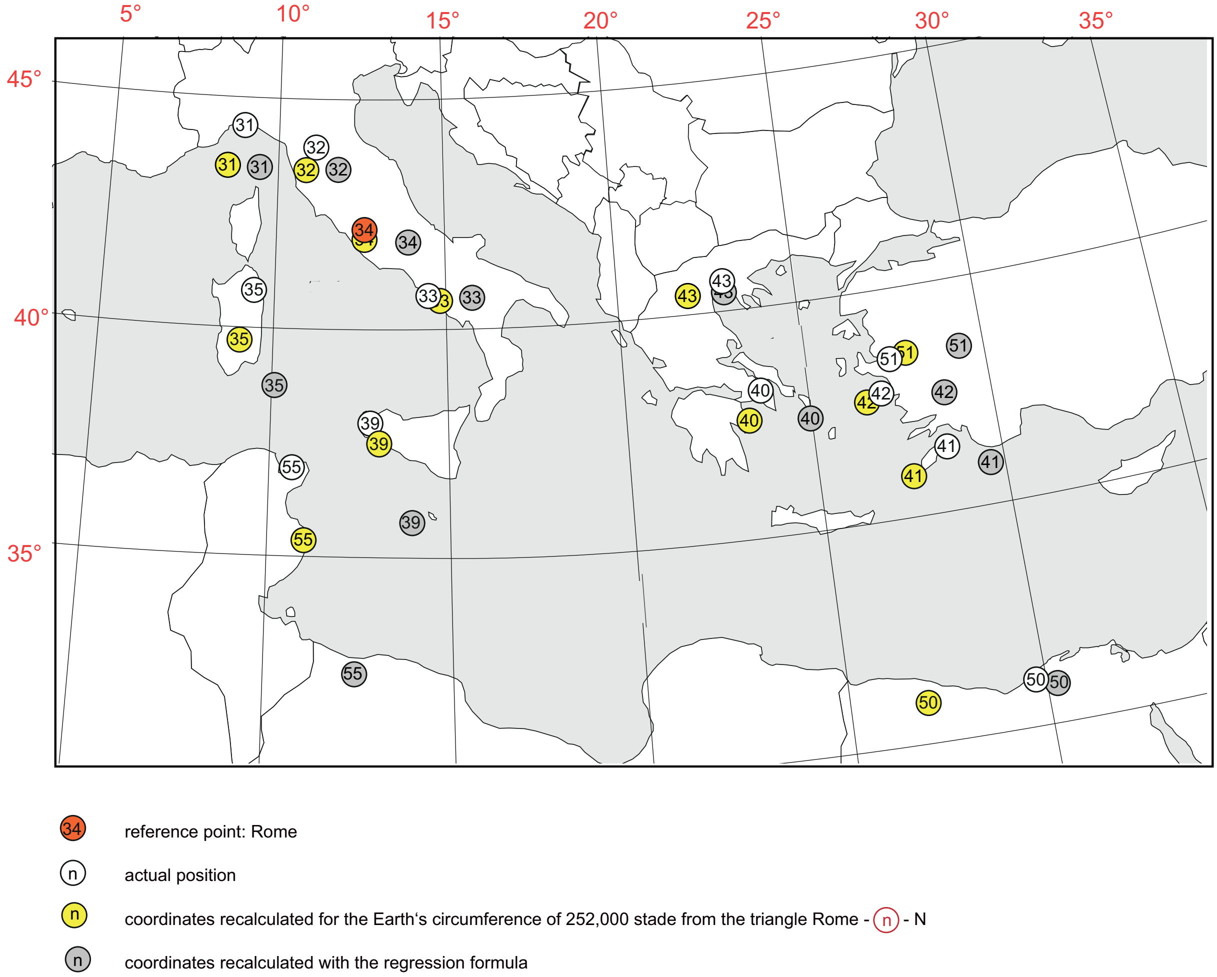

Figure 9: Recalculated locations in the eastern Mediterranean via locations obtained with the regression formula.

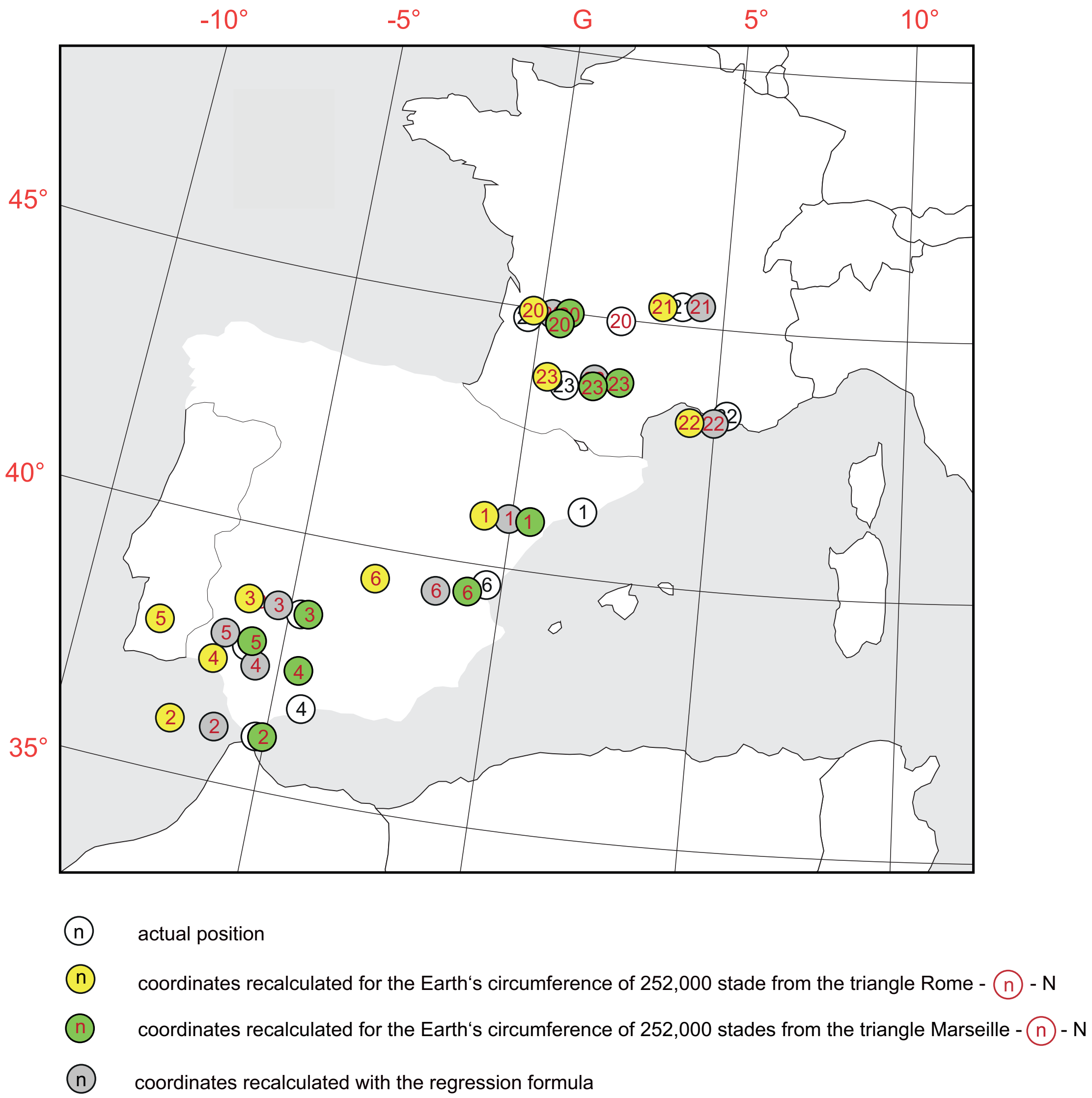

Figure 10: Recalculated locations in the western Mediterranean via locations obtained with the regression formula.

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6 Local statistical rectification of Ptolemy's maps

A thorough investigation of some local Ptolemaic maps has recently being applied for identification of the locations given in *Geography* by statistical analysis based on the Gauss-Markov model.[14] The main idea was to choose for every local map a set of identified localities with the modern positions $(\varphi^i_{modern}, \lambda^i_{modern})$ and to compare these positions with the Ptolemaic positions $(\varphi^i_{Ptolemy}, \lambda^i_{Ptolemy})$. A system of linear

[14] Kleineberg et al., 2010; Kleineberg et al., 2012.

equations connecting these coordinates was written as

$$\lambda^i_{Ptolemy} + v^i_\lambda = m_\lambda \lambda^i_{modern} + \lambda_0,$$
$$\varphi^i_{Ptolemy} + v^i_\varphi = m_\varphi \varphi^i_{modern} + \varphi_0.$$

In such a way, the systematical differences between the modern and Ptolemaic coordinates were modelled with the scale parameters m_λ and m_φ as well as with the linear terms λ_0 and φ_0. The quantities v^i_λ and v^i_φ were considered as residuals. After that, the subsets of location were chosen which have shown the same statistical behavior without noticeable systematics in residuals. Such subsets were considered to belong to the same transformation module. With the scale parameters and the linear terms being found for every module, the modern coordinates can be determined through the Ptolemaic coordinates as

$$\lambda^i_{modern} = \frac{1}{m_\lambda}\lambda^i_{Ptolemy} - \frac{1}{m_\lambda}\lambda_0,$$
$$\varphi^i_{modern} = \frac{1}{m_\varphi}\varphi^i_{Ptolemy} - \frac{1}{m_\varphi}\varphi_0.$$

In some sense, this is a more refined (and of course much more elaborated) approach than that of Russo: not only the longitudes but also the latitudes of the locations were the subject of statistical analysis. Whereas the linear terms φ_0 (the authors called them "translations") of every set can be caused by an error in determination of the latitude of a reference point used by Ptolemy to map a local district, the "translation" in longitude λ_0 (which was found to be different for different subsets) is to a great extent due to the problem of localization of the position of the Greenwich meridian for a selected region (see chapter 3). In our opinion, it would be mathematically reasonable, to correct the Ptolemaic coordinates first for the erroneously chosen circumference of the Earth and then to proceed with statistical algorithms minimizing the residuals of the errors in the local positions. At the east side of the oikoumene, where identification of the localities was always a kind of the guesswork, the methods of statistical analysis have not, in our opinion, enough of reliable input to provide a desirable output.

Unfortunately the authors have taken the elongation of Ptolemy' world map as *a priori* and also seem to have attributed as the main error the use of different values of the stade in ancient sources:[15]

> Neben systematischen Verschiebungen ist von systematischen maßstäblichenVerfälschungen auszugehen, die eher weiträumig in Ländergröße gleichartrig auftreten. Die Ursache lieg in Ptolemaios' Überschätzung der Ausdehnung der *oikoumene* in der Länge (180° bis zum östichen Ort in China statt ca. 130°) sowie darin, dass Ptolemaios mutmaßlich Unterschiede antiker Längenmaßeinheiten (Stadion-Definitionen) unwissentlich nicht berücksichtigte.

In our treatment, the simple transformation of the Ptolemaic coordinates to the circumference of the Earth measured by Eratosthenes drastically improves the positions of Ptolemaic locations – under presupposition that both scholars used the same length of the stade. It is clear that due to enormous scope of information which Ptolemy had to assimilate in his geographical treatise some pollution caused by different measuring units was unavoidable. Nevertheless, it is clear that both authors used in principle the same stade and the excessive distortion of Ptolemy's world word is due to a wrongly conceived standardized degree. Our thesis presents an easy explanation and a coherent correction for this.

7 Conclusion

The results presented in our text have not, at first, aimed at obtaining a new estimation of the length of the stade used by Eratosthenes or by Ptolemy. Being rather motivated by a purely mathematical problem, by recalculating the spherical coordinates given at the sphere with the circumference of 180,000 units to the spherical coordinates at the sphere with the circumference of 252,000 units, we have obtained

[15] Kleineberg et al. 2012, p. 13.

results which show that the excessive distortion of Ptolemy's world map can be easily explained under the presupposition that the value for the circumference of the Earth used by Ptolemy is expressed in the same metrical value of the stade which Eratosthenes used. In fact, Ptolemy's world map is distorted in a very special way: the latitudes of the known localities coincides approximately with their actual values whereas the the longitudes show an excessive distortion in the east-west direction. Our recalculations show that such a distortion appears due to an erroneously adopted value for the circumference of the Earth or, equivalently, for the length of $1^\circ = 500$ stades at the equator used by Ptolemy instead of $1^\circ = 700$ stades as given by Eratosthenes.

The first results for the Mediterranean region show that if Ptolemy would have adopted Eratosthenes' value, his map would have an unexpectedly high level of precision, in many cases, the longitudinal errors would be comparable with the latitudinal ones. As a consequence, by linking the recalculated positions of the identified localities with their actual positions, we can confirm a very high precision of Eratosthenes' result for the circumference of the Earth. The result is supported independently by Russo's statistical estimation of the length of the stade as 155.6 meter.

The method presented in our text allows

- to restore in many cases Ptolemy's raw data, that is, the distances between the localities and the directions of the routes;
- to perform a global recalculation of the Ptolemaic positions for a corrected circumference of the Earth;
- to choose an arbitrary reference point for linking the positions with the position of the Greenwich meridian;
- to identify the localities which could not be linked with a chosen reference point because of the erroneously adopted circumference of the Earth;
- to determine the local centers for the measuring of the distances incorporated in Ptolemy's data base of distances.

Let us stress here that the results which we have obtained with our recalculations can locally be interpreted as a deformation of Ptolemy' maps caused by to the different definitions of the stades used by both scholars. But to create a world map globally, Ptolemy should have ascribed to the metrical value of a stade some angular value gained from the measuring of the circumference of the Earth. And here, as we have shown, he made a mistake.

We do hope that the proposed interlink between the definition of the circumference of the Earth and the geographical mapping performed by Ptolemy will throw some new light on this chapter of the ancient geography.

8 Appendix A

The Greeks expressed long-range distances normally (but far from exclusively) in stadia or stades. Like other Mediterranean oder Near Eastern units for measuring distance, the Greek units were originally based on lengths of the human body (cf. Vitr. 3.1.5). Thus, a Greek stadion is equal to 600 feet (or 6 plethra or 100 fathoms), six feet equal four cubits, four palms equal one foot, and six palms equal one cubit (Hdt. 2.149.3). Since the human body can vary within certain limits, there were a bewildering diversity for ancient units of measurement. Likewise the racetracks found and excavated in Greek cities are all different. The fact that it is impossible to attain an accurate modern equivalent to the Greek stade is obfuscated even in standard reference works and secondary works where, e.g. by Pochan 1932/33: 277–314; cf. H. von Mžik 1933: 105–112: 158.57 m), a stadion is rendered as exactly 158.314 m! Such a precision existed nowhere in antiquity. Nevertheless, in order to ensure reliable exchanges and transactions, there must have been some efforts to standardize and publicize the ratios between the more widely recognized standards like the Egyptian royal cubit, the Babylonian cubit, the Doric and Attic foot etc. The German scholar Lehmann-Haupt (1929: 1931–1963) tried to show that there were at least seven different stadia in use during Greek times. Accordingly, modern scholars have hypothized about the length of Eratostheness stadium, ranging between 148.5 and 210 m. Especially the extreme values of 148 and more than 180 m are quite implausible for various kinds of reason. A stadion of 1/10 of a Roman mile (= 500 feet = 148.5 or 148.8 m) is not attested for Classical or Hellenistic times (but see schol. Lucian. Icarom. 24, p. 99, 10sq. Rabe citing "others"]; Itin. Burdigal. p. 609, 4 Wesselingius (= p. 100 Cuntz); more in Prell 1956/57: 561, 562). Nevertheless many scholars have adopted this view (see already dAnville 1759: 82–91, 92–100; id. 1769: 69–89, 181–2; Lehmann-Haupt 1929: 1952–1960; Prell 1956/57: 549–563; Fischer 1975: 159–160). The Attic-Roman stade, also called "stadium Olympicum", of 185 m (400 cubits of 0.462 m equal to 185 m, in other words, a Roman mile comprises of 8 1/3 stades) is also often attributed to Eratosthenes by modern scholars (see Columba 1895: 63–68; Dreyer 1914: 353; Czwalina 1925: 295; Dicks 1960: 42–46; Rawlins 1981: 218; Pothecary 1995: 49-67. Cf. also Cimino 1982: 1121 and Engels 1985: 298–311. Cimino has exactly $184,8$ m, Engels $184,88$ m for the Olympic stade. For a similar result ($184,8$ m) see Dewald 1995: 155-160, who derived the different lengths of the stade from different degrees of latitude. But his collection of ancient sources is quite arbitrarily. Following Columba 1895, Manna 1986: 41–42, derives: $4517.6 : 2\pi R = 7°30' : 360°$, where π is 3 1/7 and the "Alexandrian" stade is 185.5 m)[16] because it was in use in Egypt. But in this *Geography* Eratosthenes relied rarely on "Egyptian" sources and drew mostly upon older Greek sources. It seems utterly incredible that he converted the received data to a more "modern" or "local" unit. A stade unit of 210 m (favoured as early as Abendroth 1866: 34–35) is based on the Royal Egyptian cubit (525 mm), which was used for many edifices. According to Herodotus (2.168) the Egyptian cubit was as long as the "Samian" one, which, multiplied by 400, produced the "Ionian" stade. This stade unit also seems very improbable as Eratosthenes' standard unit. Not attested and therefore implausible are also units of 168 m (Thalamas 1921: 158–159. Gulbekian 1987: 362–3, reckons with a stade of 166.7 m, i.e. $840\,\mathrm{km} * 103/5.040$ stades. At the heart of Gulbekian's thesis lies a precision of measurements which was impossible to achieve for ancient engineers) and 177.6 m (Nissen 1903: 241; Janvier 1993: 13, 20–21 (600 feet à 296 mm); Reymond 1924: 82, has 177.4 m). Of all the different stade units, the most likely one for Eratosthenes is that of c. 157.5 or 157.7 m. This hypothesis can either point to ancient sources and match well with modern averaging methods. For the ancient sources, see especially Plin. nat. 12.53: *schoenus patet Eratosthenis ratione stadia XL, hoc est p. V* (but later [nat. 2.247; cf. Vitr. 1.6.9] Pliny compares 252000 stades to 31500000 milia passuum, i. e. 46620 km, which would alllude to a stade of 185 m).[17] A so-called Egyptian *schoenus* equals 12000 cubits of 0.525 m or 6300 metres, which can be converted to a stade of 300 cubits or 157.5 m. This view is adopted among others by Letronne 1851: 104–119, 212–246; Hultsch 1882: 60–63; Müllenhoff 1890, I, 259–296; Tannery 1893: 109–110; Hultsch 1897: 292; Dreyer 1953: 175; Viedebantt 1915: 232–252, id. 1920; 94–109; Miller 1919: 6–7; Oxé 1963: 269–270; Aujac 1966: 176–179; Fraser 1972: II, 599, n. 312; Mouraviev, 1986/88: 235–247; Stückelberger 1988: 188; Dutka 1993/94: 63–4; Meuret 1998, 163–4. Nevertheless, it must be pointed out that such a stade unit is attested in ancient sources only indirectly. In an interesting article, the Russian scholar Firsov (1972: 154–174) has averaged 81 measurements of Eratosthenes, mostly transmitted by Strabon. He also contends that the stade in Hellenistic times was normalized due to practical reasons. Modern opinions according to which the stade varied according to certain times and places are unwarranted in his view (see also Engels 1985: 306–307).[18] While his hypothesis may be true, they are unable to prove stringently that a) Eratosthenes while assembling a vast amount of measurement data in ancient sources, relied exclusively on one stade unit (or, alternatively converted all data into "his" hypothetical stade unit) nor that Eratosthenes' stade measured exactly c. 157.5 m. Despite all this problems, this seems to be most plausible solution. Strangely enough, we hear next to nothing about the "different" stadia

[16] Lepsius 1877: 3–8 (multiples of the "little" cubit) and Ruiz Cadalso, 1938, 148–157. have the slight variant of 180 m. Cf. also Lloyd 1973: 50 (1/8 of a Roman mile, which means that the circumference of the Earth was c. 17% too high).

[17] For the *schoinos* in Eratosthenes see in addition to Plin. nat. 12.53 also Strab. 15.1.11, C 689 (v. l.) and Arr. Ind. 3.1 with the commentary of Berger 1880: 225, n. 4.

[18] A slightly different view of 158.314 m is given by Pochan 1932/43, 277–314; H. von Mžik calculates 158.57 m in 1933: 105–112.

in ancient sources. While assembling ancient measurement data, no ancient geographer bothered to lend thoughts to the problem that the measurements he drew upon in his various sources could have been, and surely were, taken with different lengths of a stade. It is hard to think that the ancients were not aware of this problem. That they did not care probably shows that they considered it insignificant or insolvable. On the one hand, they had normally no means to ascertain which stade has been used for certain regions and times, on the other hand, the extreme values for a stade (e.g. 148.8 or more than 180 metres) played no major role *in praxi*. The ancient merchants at least - and it were the merchants data the ancient geographers relied upon - had a pretty good notion in mind how long a stade was.

9 Appendix B

The numerical results in the following tables gives pairs of latitude and longitude coordinates according to the standard agreement: longitudes are expressed positively in the eastern direction of the Greenwich meridian and negatively in the western direction and the latitudes are positive in the northern direction relative from the Earth's equator.

Table 1: Recalculation of Ptolemy's geographical coordinates for the Earth's circumference of 252,000 stades.
Region: eastern Europe. Reference point: Alexandria

City number Name	Ptolemy's coordinates φ; λ	Ptolemy's coordinates relative to Greenwich	recalculated coordinates	modern coordinates
30: Ancona	43.6700; 36.5000	43.6700°; 5.9035°	43.6700°; 16.9906°	43.6160° ; 13.5208°
31: Genua	42.8300; 30.0000	42.8300°; −0.5965°	42.8300°; 10.8397°	44.4070°; 8.9340°
32: Florence	43.0000; 33.8300	43.0000°; 3.2335°	43.0000°; 14.1122°	43.7714°; 11.2539°
33: Naples	40.5000; 40.0000	40.5000°; 9.4035°	40.5000°; 17.8081°	40.8550°; 14.2590°
34: Rome	41.6700; 36.67000	41.6700°; 6.0735°	41.6700°; 15.6903°	41.8925°; 12.4844°
35: Olbia	38.7500; 31.6667	38.7500°; 1.0702°	38.7500°; 10.4482°	40.9167°; 9.5000°
36: Syrakus	37.2500; 39.5000	37.2500°; 8.9035°	37.2500°; 15.8848°	37.0833°; 15.2833°
37: Messina	38.5000; 39.5000	38.5000°; 8.9035°	38.5000°; 16.3574°	38.1833°; 15.5500°
38: Mount Etna	38.0000; 39.0000	38.0000°; 8.4035°	38.0000°; 15.7675°	37.7294°; 15.0002°
39: Marsala	36.0000; 37.0000	36.0000°; 6.4035°	36.0000°; 13.6724°	37.8000°; 12.4333°
40: Athens	37.2500; 52.7500	37.2500°; 22.1535°	37.2500°; 28.2775°	37.9700°; 23.7360°
41: Rhodes	36.0000; 58.6700	36.0000°; 28.0735°		36.4402°; 28.2108°
42: Samos	37.5800; 57.0000	37.5800°; 26.4035°		37.7581°; 26.9744°
43: Thessaloniki	40.3300; 49.8300	40.3300°; 19.2335°	37.7643°; 22.5467°	40.6300°; 22.9490°
50: Alexandria	31.00000; 60.5000	31.0000°; 29.9035°	reference point	31.1590°; 29.9035°
51: Smyrna	38.5800; 58.4200	38.5800°; 27.8235°		38.4138°; 27.1468°
55: Carthage	32.6700; 34.8300	32.6700°; 4.2335°	32.6700°; 11.6412°	36.8580°; 10.3310°

Table 2: Recalculation of Ptolemy's geographical coordinates for the Earth's circumference of 252,000 stades.
Region: eastern Europe. Reference point: Rome

City number Name	Ptolemy's coordinates φ; λ	Ptolemy's coordinates relative to Greenwich	recalculated coordinates	modern coordinates
30: Ancona	43.6700; 36.5000	43.6700°; 12.3144°	not possible	43.6160° ; 13.5208°
31: Genua	42.8300; 30.0000	42.8300°; 5.8144°	42.8300°; 7.8488°	44.4070°; 8.9340°
32: Florence	43.0000; 33.8300	43.0000°; 9.6444°	43.0000°; 10.8942°	43.7714°; 11.2539°
33: Naples	40.5000; 40.0000	40.5000°; 15.8144°	40.5000°; 14.6003°	40.8550°; 14.2590°
34: Rome	41.6700; 36.67000	41.6700°; 12.4844°	reference point	41.8925°; 12.4844°
35: Olbia Case 2	38.7500; 31.6667	38.7500°; 7.4811°	38.7500°; 10.1173° 39.6060°; 8.8670°	40.9167°; 9.5000° 40.9167°; 9.5000°
36: Syrakus	37.2500; 39.5000	37.2500°; 15.3144°	38.5197°; 14.5419°	37.0833°; 15.2833°
37: Messina	38.5000; 39.5000	38.5000°; 15.3144°	39.4126°; 14.5324°	38.1833°; 15.5500°
38: Mount Etna	38.0000; 39.0000	38.0000°; 14.8144°	39.0533°; 14.1738°	37.7294°; 15.0002°
39: Marsala	36.0000; 37.0000	36.0000°; 12.8144°	37.62001°; 12.7254°	37.8000°; 12.4333°
40: Athens	37.2500; 52.7500	37.2500°; 28.5644°	37.2500°; 23.2376°	37.9700°; 23.7360°
41: Rhodes	36.0000; 58.6700	36.0000°; 34.4844°	36.0000°; 27.3238°	36.4402°; 28.2108°
42: Samos	37.5800; 57.0000	37.5800°; 32.8144°	37.5800°; 26.5036°	37.7581°; 26.9744°
43: Thessaloniki Case 2	40.3300; 49.8300	40.3300°; 25.6444°	40.3300°; 21.7974° 40.8650°; 21.9433°	40.6300°; 22.9490° 40.6300°; 22.9490°
50: Alexandria	31.00000; 60.5000	31.0000°; 36.3144°	31.0000°; 26.6977°	31.1590°; 29.9035°
51: Smyrna	38.5800; 58.4200	38.5800°; 34.2344°	38.5800°; 27.7394°	38.4138°; 27.1468°
55: Carthage	32.6700; 34.8300	32.6700°; 4.2335°	35.2442°; 11.1271°	36.8580°; 10.3310°

Table 3: Recalculation of Ptolemy's geographical coordinates for the Earth's circumference of 252,000 stades.
Region: western Europe. Reference point: Rome

City number Name	Ptolemy's coordinates φ; λ	Ptolemy's coordinates relative to Greenwich	recalculated coordinates	modern coordinates
1: Barcelona	41.0000; 17.2500	41.0000°; −6.9356°	41.0000°; −1.3585°	41.3833° ; 2.1833°
2: Calpe/Gibraltar	36.2500; 7.5000	36.2500°; −16.6856°	36.2500°; −7.7196°	36.1258°; −5.3431°
3: Cordoba	38.0800; 9.3300	38.0800°; −14.8556°	38.0800°; −6.7264°	37.8833°; −4.7667°
4: Malaga	37.5000; 8.8300	37.5000°; −15.3556°	37.5000°; −6.9932°	36.7194°; −4.4200°
5: Seville	37.8300; 7.2500	37.8300°; −16.9356°	37.8300°; −8.1857°	37.3772°; −5.9869°
6: Valencia	39.0000; 14.0000	39.0000°; −10.1856°	39.0000°; −3.4983°	39.4667°; −0.3833°
34: Rome	41.6700; 36.67000	41.6700°; 12.4844°	reference point	41.8925°; 12.4844°
20: Bordeaux	45.0000; 18.0000	45.0000°; −6.1856°	45.0000°; −0.4444°	44.8386°; −0.5783°
21: Lyon	45.8300; 23.2500	45.8300°; −0.9356°	45.8300°; 3.7964°	45.7597°; 4.8422°
22: Marseille	43.0800; 24.5000	43.0800°; 0.3144°	43.0800°; 3.8987°	43.2964°; 5.3700°
23: Toulouse	44.2500; 20.1700	44.2500°; −4.0156°	44.2500°; 0.9705°	43.6044°; 1.4439°

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