From Einstein to Schücking: Mach's Principle and Relativistic Cosmology

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On Solving Einstein's Gravitational Field Equations

Einstein's Gravitational Field Equations from November 1915:

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}$$

wo $\quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$\kappa = -8\pi\, G$ (G = Newton Constant)

Einstein vs. Newton

NEWTON

$$\varrho$$

$$\downarrow$$

$$\Delta\Phi = 4\pi G\varrho$$

$$\downarrow$$

$$\Phi$$

$$\downarrow$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{\mathrm{d}\Phi}{\mathrm{d}r}$$

(Bewegungsgleichung)

EINSTEIN

$$\varrho$$

$$\downarrow$$

$$T_{ik}$$

$$\downarrow$$

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\kappa T_{ik}$$

$$\downarrow$$

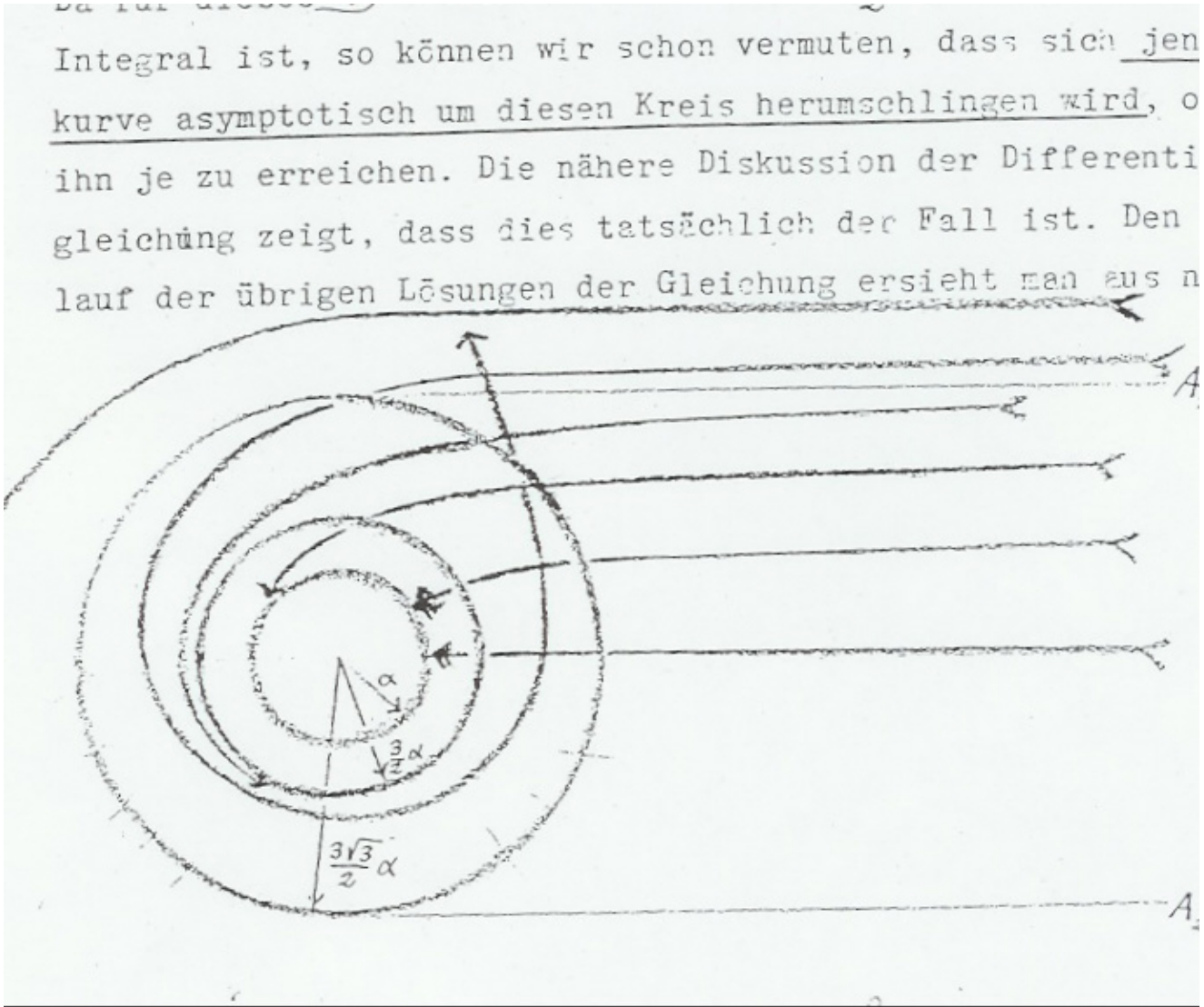
$$g_{ik}$$

$$\downarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(g_{ik}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^k}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}$$

(geodätische Linie im Riemannschen Raum)

Before Black Holes: Hilbert's Version of the Schwarzschild Metric

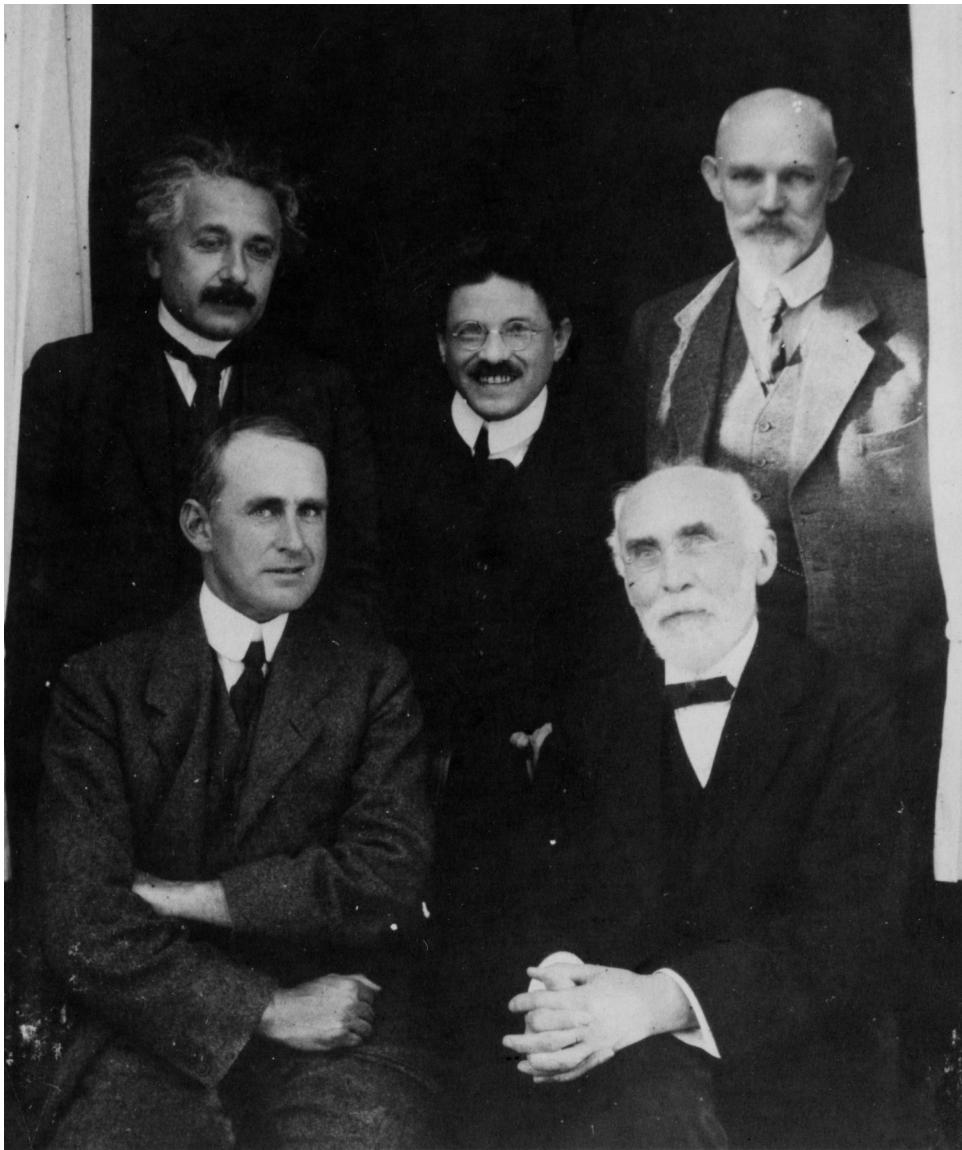


Relativistic Cosmology: Background Events

Conversations in Leiden,
1916-1917

Group Photo in Leiden, 1921

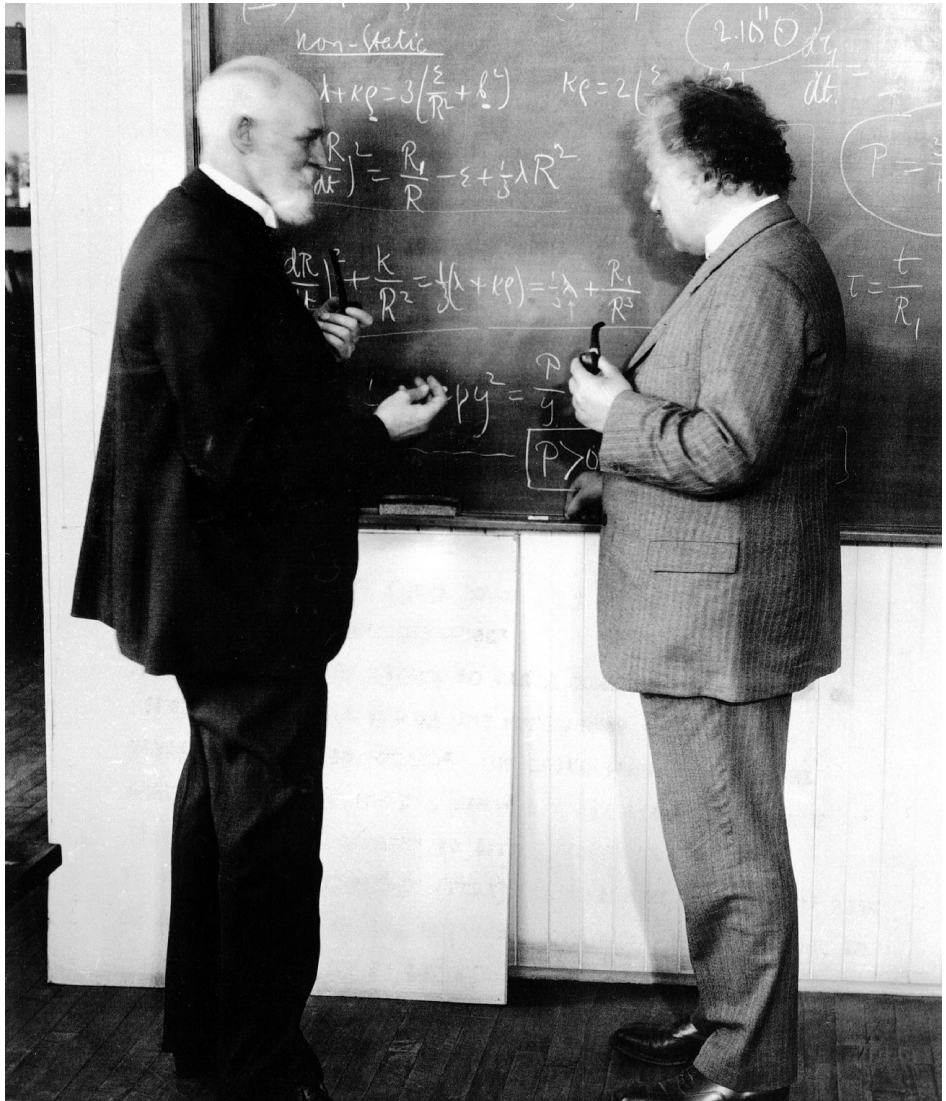
- A. S. Eddington, H. A. Lorentz (front)
- Einstein, Ehrenfest, de Sitter (back)



Albert Einstein and Willem de Sitter

- Fall 1916: **first meeting with de Sitter**
- Leiden's astronomer **a close associate of Lorentz und Ehrenfest**
- By 1916 all three were deeply engaged in Einstein's new **generalized theory of relativity**
- But **de Sitter had a head start** having studied the **earlier version of Einstein and Grossmann from 1913!**
- In 1916 **de Sitter corresponded with Einstein about gravitational radiation!**

Cosmological Combatants: De Sitter vs. Einstein



De Sitter's Early Critique and Einstein's Reaction

Einstein to de Sitter, 2 February 1917

"Presently I am writing a paper on the **boundary conditions in gravitation theory**. I have completely **abandoned my idea on the degeneration of the** $g_{\mu\nu}$ **'s, which you rightly disputed.** I am curious to see what you will say about the rather outlandish conception I have now set my sights on."

Einstein to Paul Ehrenfest, 4 February 1917

“I have again perpetrated something in gravitational theory that puts me in a bit of danger of being committed to a madhouse. I hope you don’t have any in Leiden so that I can visit you again safely.”

Einstein's Cosmological Field Equations from 1917

The cosmological equations are:

$$G_{\mu\nu} - \lambda g_{\mu\nu} = T_{\mu\nu}$$

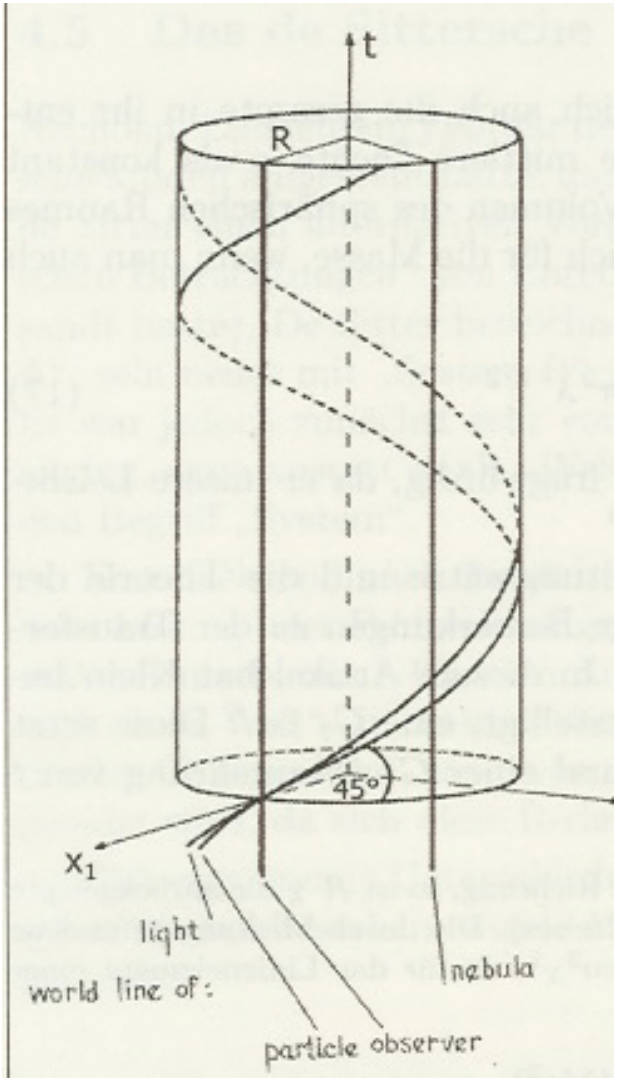
where the tiny constant λ corresponds to a small repelling force.

On Avoiding Boundary Conditions at Spatial Infinity

- **Einstein and Jakob Grommer** had tried to find **global solutions** of the original field equations.
- Solutions were to be **centrally symmetric, static,** and with **appropriately chosen boundary values at spatial infinity.**
- **DeSitter criticized this approach**
- Einstein went on to conclude that **space must be finite, but unbounded.**
- By **adding the cosmological constant** Einstein came up with his **cylindrical universe.**

The „Einstein Universe“, 1917

- Einstein modified his field equations for gravitation to produce **a Machian universe**
- **This „model“ represented a static universe compatible with general relativity**
- Felix Klein dubbed it **Einstein ‘s „Cylindrical World“**



De Sitter Announces an Alternative Cosmology

De Sitter to Einstein, 20 March, 1917

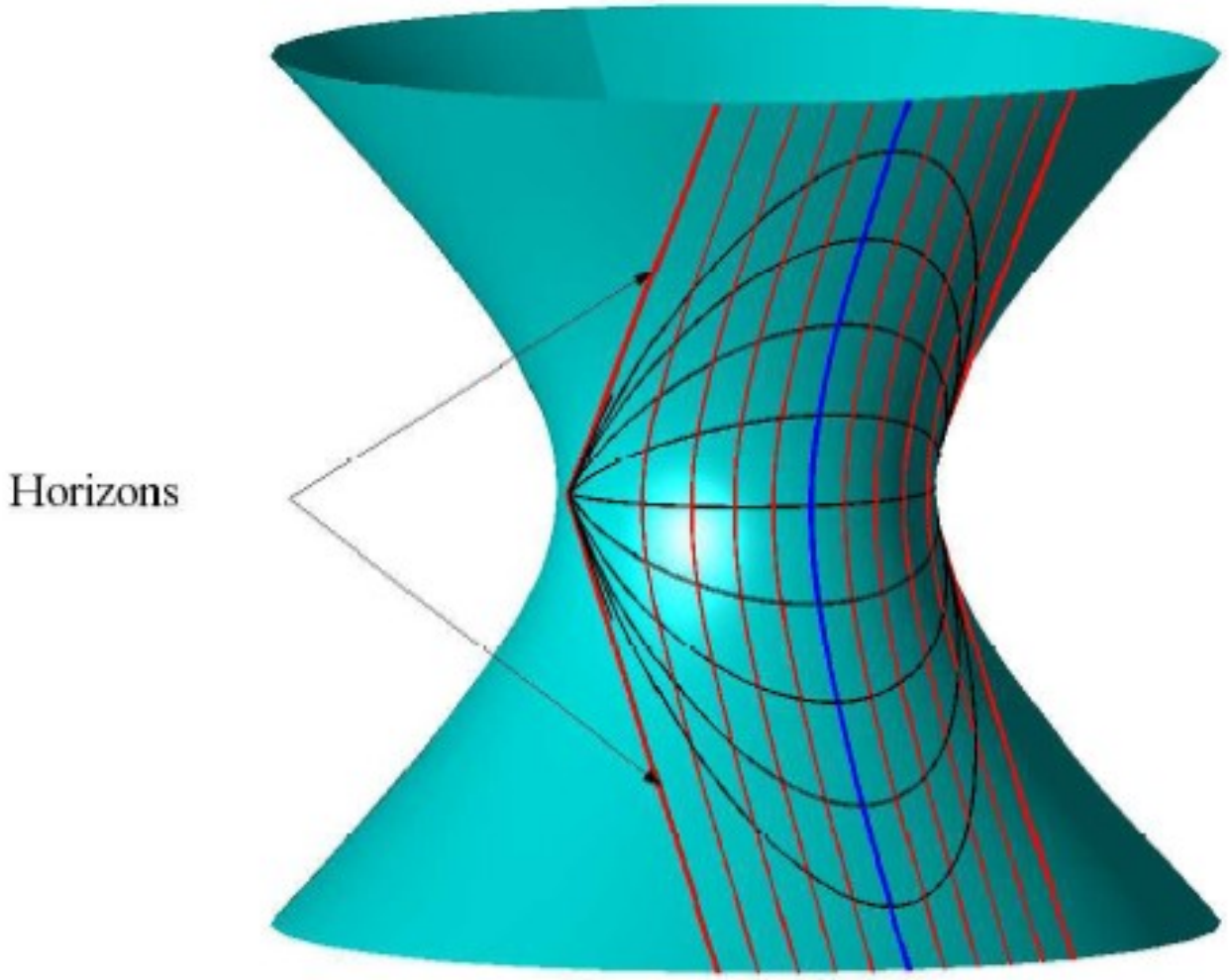
"I have found that the equations

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 0,$$

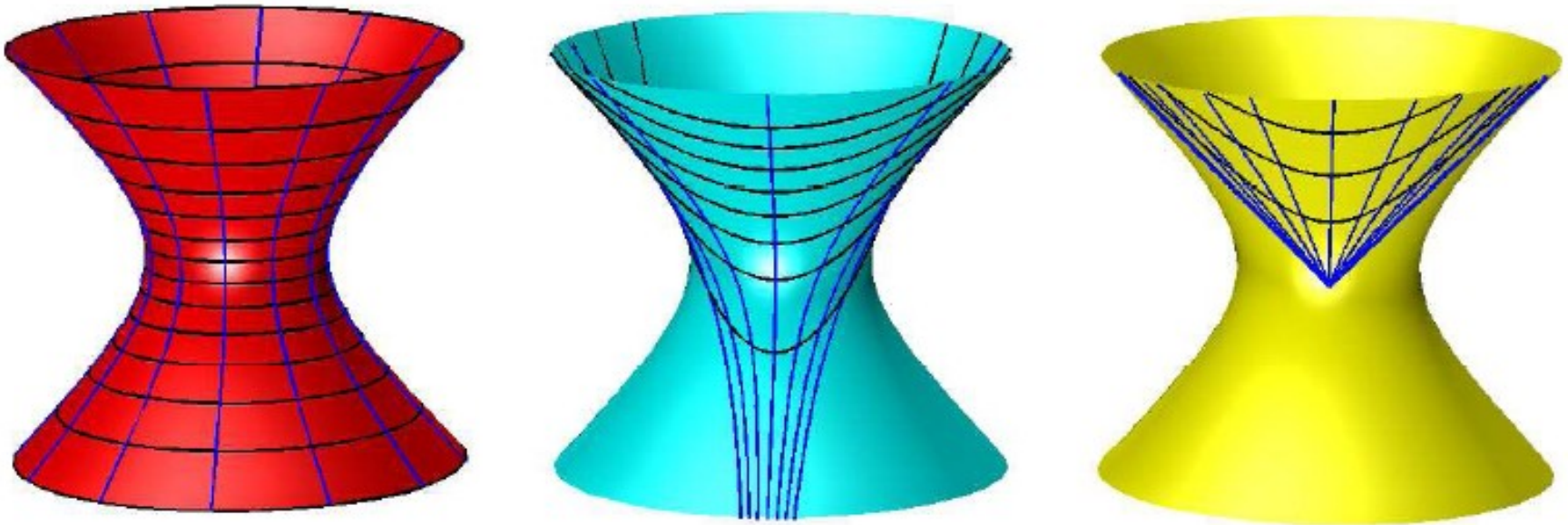
thus your equations *without matter*, can be satisfied by [the de Sitter metric] . . .."

"I do not know if it can be said that 'inertia is explained' in this way. I do not concern myself with explanations. If a single test particle existed in the world, where there was *no* sun and stars, it would have inertia."

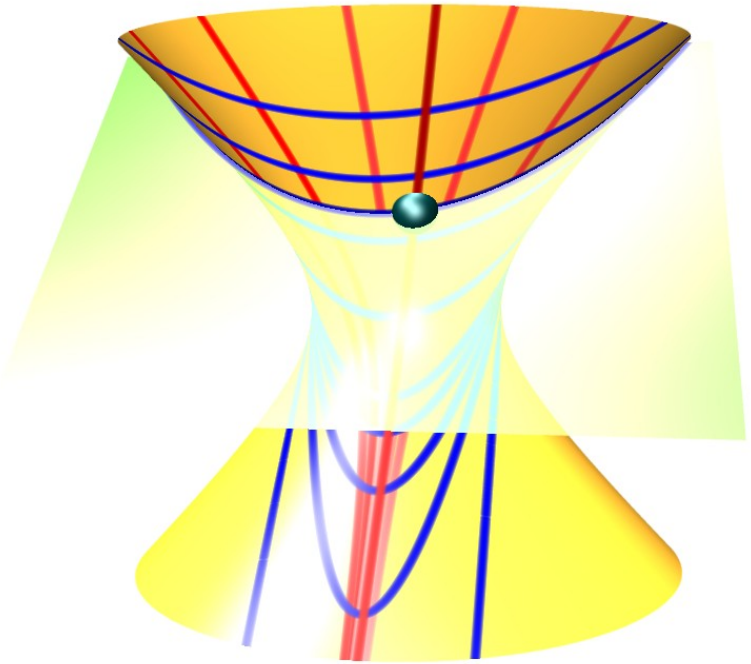
Modern Picture of de Sitter ‘s World with Static Coodinates

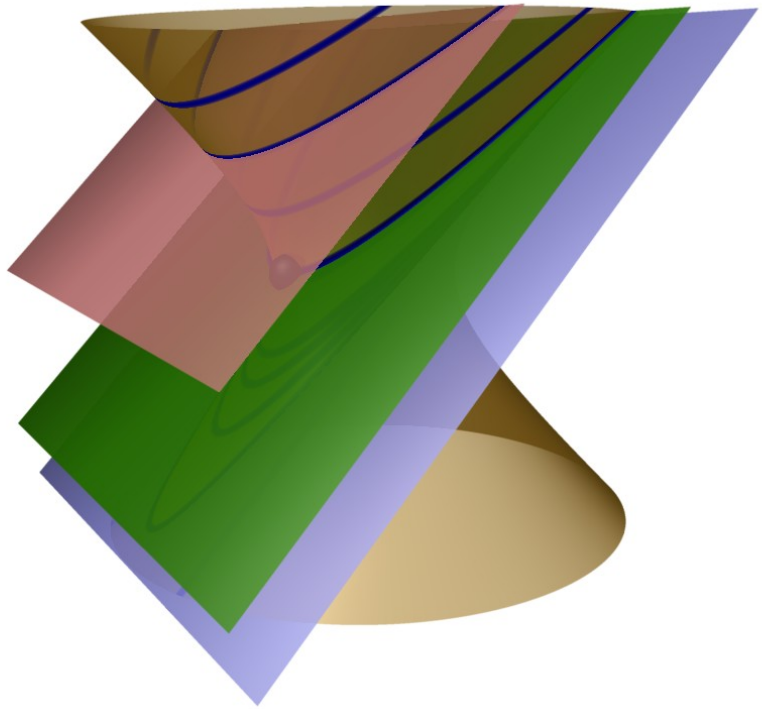


De Sitter Spacetime admits Spatial Sections of Positive, Zero, and Negative Curvature



How a de Sitter Model can reveal an „Expanding Universe“





Taking a Glance Backward: Historical Background

On Machian Ideas in Einstein's Theory of Gravitation

Two Early Theoretical Contexts

- Original context: Mach's critique of **Newtonian mechanics**
- New context: **field physics**

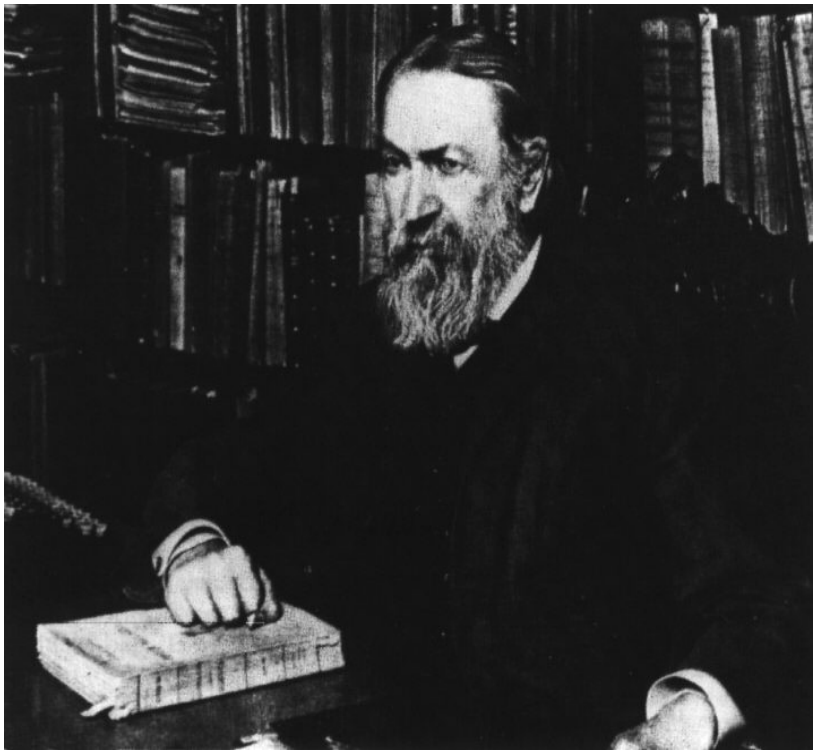
  1) Einstein's appeal to the principle of relativity in electrodynamics;

  2) coupling of mass and energy in SR;

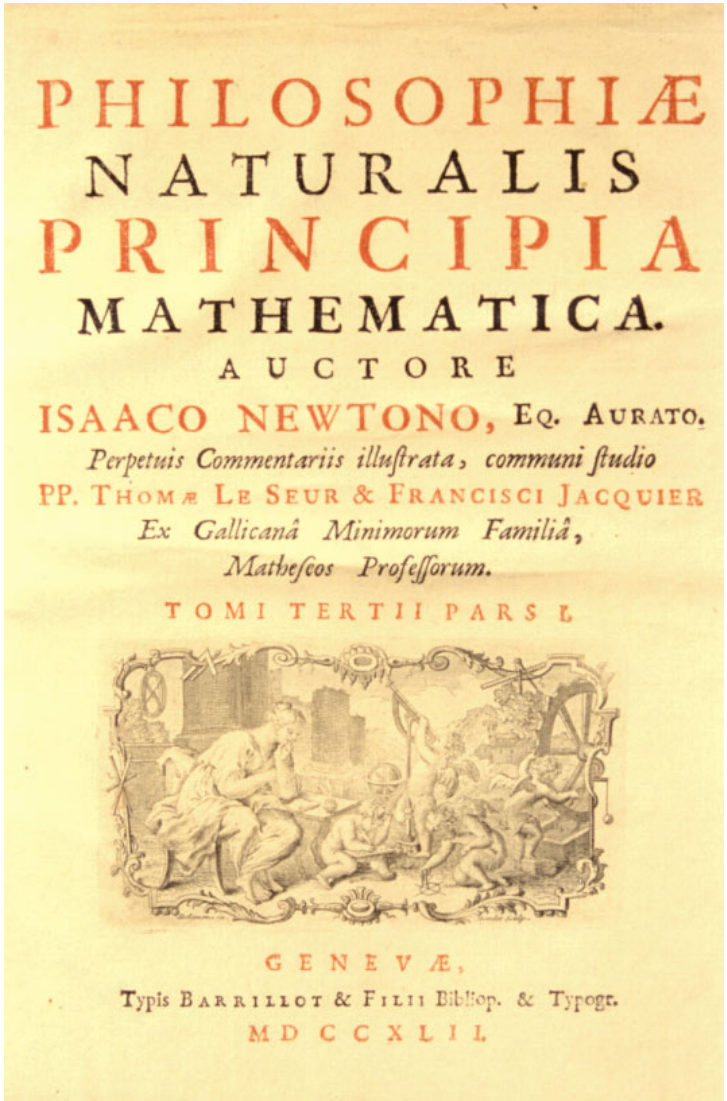
  3) coupling of inertia and gravity in GR

Mach's Critique of Newtonian Mechanics

- Ernst Mach famously rejected Newton's notions of **absolute space** and **absolute time**
- **Inertial properties of matter** were **due to some kind of interaction with distant masses** (the fixed stars)



Newton's *Principia* (1687)



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.
AUCTORE
ISAACO NEWTONO, Eq. Aurato.
Perpetuis Commentariis illuſtrata, communi ſtudio
PP. Thomæ Le Seur & Francisci Jacquier
Ex Gallicanâ Minimorum Familiâ,
Matheſeos Profeſſorum.
TOMI TERTII PARS I.

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Typis Barrillot & Filii Bibliop. & Typogr.
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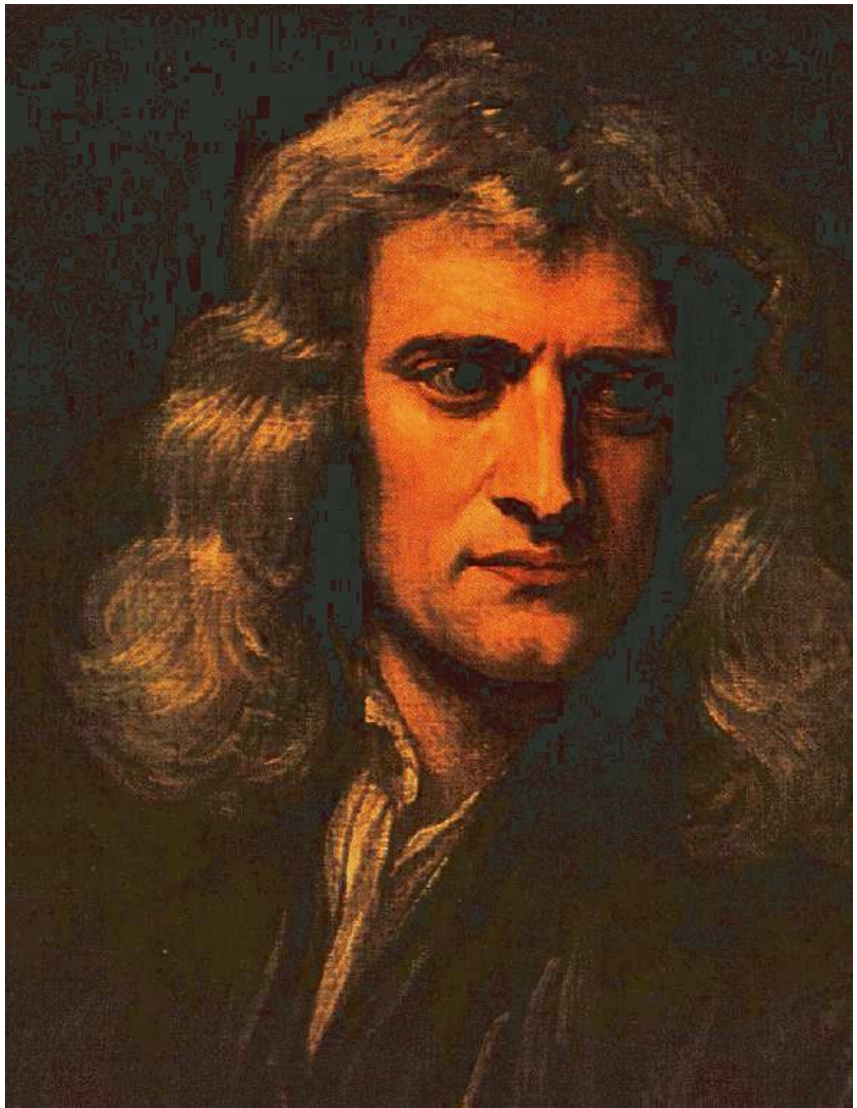
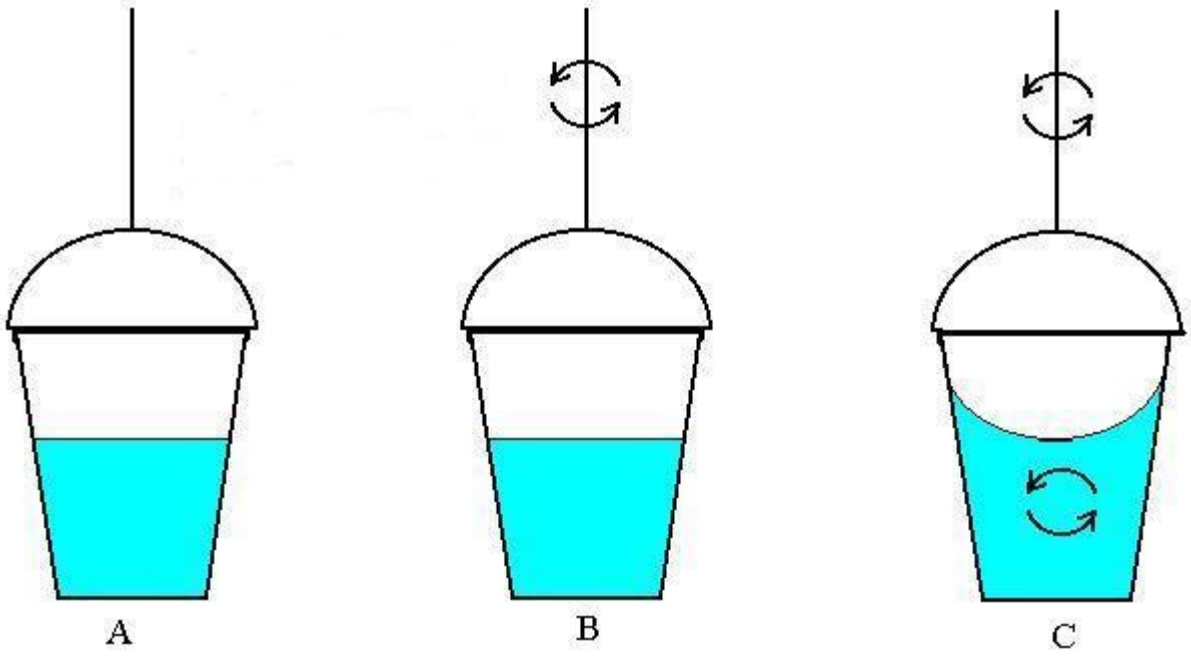


Newton’s Rotating Water Bucket



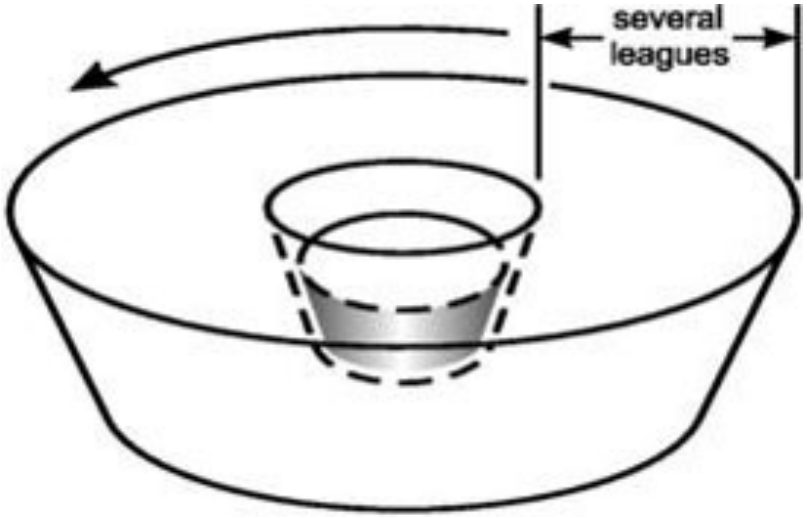
Eimer und Wasser in Ruhe (im Raum und zueinander)

beginnende Rotation des Eimers; Wasser noch in Ruhe (Relativbewegung zw. Eimer und Wasser)

Rotation von Eimer und Wasser im Raum (keine Relativbewegung zw. Eimer und Wasser)

Mach's Influence on Einstein

- **Problem of Rotation** played a key role in Einstein's efforts to **extend the principle of relativity to non-inertial frames**
- In SR he showed how **small inertial effects could be produced by rotating masses**

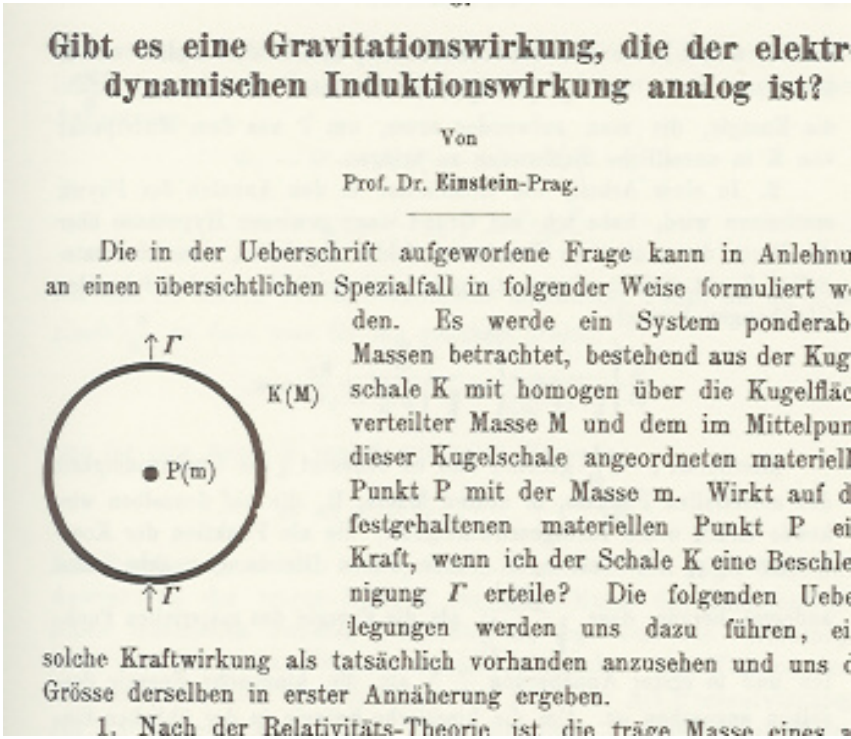


“Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?”

Einstein ‘s first attempt to implement a Machian Program, 1912

Einstein in pursuit of a Machian approach to Gravitation (1912)

- 1913: **Einstein writes to Mach about the relativity of inertia**
- 1918: Hans Thirring carries out similar ideas in GR



Gibt es eine Gravitationswirkung, die der elektr dynamischen Induktionswirkung analog ist?

Von

Prof. Dr. Einstein-Prag.

Die in der Ueberschrift aufgeworfene Frage kann in Anlehnu an einen übersichtlichen Spezialfall in folgender Weise formuliert w den. Es werde ein System ponderab Massen betrachtet, bestehend aus der Kug schale K mit homogen über die Kugelfläc verteilter Masse M und dem im Mittelpun dieser Kugelschale angeordneten materiell Punkt P mit der Masse m. Wirkt auf d festgehaltenen materiellen Punkt P ei Kraft, wenn ich der Schale K eine Beschle nigung Γ erteile? Die folgenden Ueb legungen werden uns dazu führen, ei solche Kraftwirkung als tatsächlich vorhanden anzusehen und uns Grösse derselben in erster Annäherung ergeben.

1. Nach der Relativitäts-Theorie ist die träge Masse eines

Einstein to Mach, June 1913

"**it... turns out that inertia originates in a kind of interaction between bodies, quite in the sense of your considerations on Newton's pail experiment...** If one rotates [a massive shell of matter] relative to the fixed stars about an axis going through its center, a Coriolis force arises in the interior of the shell; that is, the plane of a Foucault pendulum is dragged around (with a practically unmeasurably small angular velocity)."

Machian Motifs in Relativistic Cosmology

Einstein's "Cosmological Considerations" (1917)

"In a consistent theory of relativity there can be no inertia *relative to 'space,'* but only an inertia of masses *relative to one another.* If, therefore, I have a mass at a sufficient distance from all other masses in the universe, its inertia must fall to zero."

Willem de Sitter as Skeptic

- De Sitter took a skeptical view of Mach's principle, which he saw as a metaphysical idea rather than a testable scientific hypothesis
- He also saw himself as differing in this respect from Einstein, to whom he wrote: "Our differences in belief amount to this: **whereas you have a definite belief, I am a skeptic**." (18 April, 1917)

Einstein on Mach's Principle, 1918

Einstein on Mach's Principle, 1918

- Einstein explicitly articulated what he understood by Mach's Principle in a short note, published in *Annalen der Physik* in May 1918
- Philosophers of science know this paper as **Einstein's reply to Kretschmann**, who claimed **general covariance was a purely formal principle without physical significance** (a point Einstein conceded)
- Einstein replied that **GR was a theory about space-time coincidences**
- **But what about Mach's Principle?**

Einstein's Machian Views, ca. 1918

- In his 1918 note, Einstein named three principles as foundations for GR:

  a) Principle of Relativity,

  b) Principle of Equivalence,

  c) **Mach's Principle: the G-field is completely determined by the energy tensor of matter**
- In a footnote, he stated that until now he had not clearly separated a) and c), the latter being **a generalization of Mach's ideas about the origin of inertia**

Status of the Three Principles

- a) just asserts that laws of physics can be expressed in terms of generally covariant equations, **a useful heuristic guideline**
- b) was central to Einstein's approach from 1907
- c) **"is a different story. The necessity to uphold it is by no means shared by all colleagues, but I myself feel it is absolutely necessary."**
- The remainder of the paper explains how c) led him to modify the field equations by introducing the cosmological constant, etc.

Mach's Principle: the Context of Articulation

- Einstein wrote this reply to Kretschmann's critique on 6 March, 1918
- A closer reading indicates that he mainly used it to present arguments **bolstering his case for Mach's principle and its relevance for cosmology**
- **Precisely one day later,** on 7 March, he submitted a short notice to the Prussian Academy entitled **"Kritisches zu einer von Hrn. De Sitter gegebenen Lösung der Gravitationsgleichungen"**

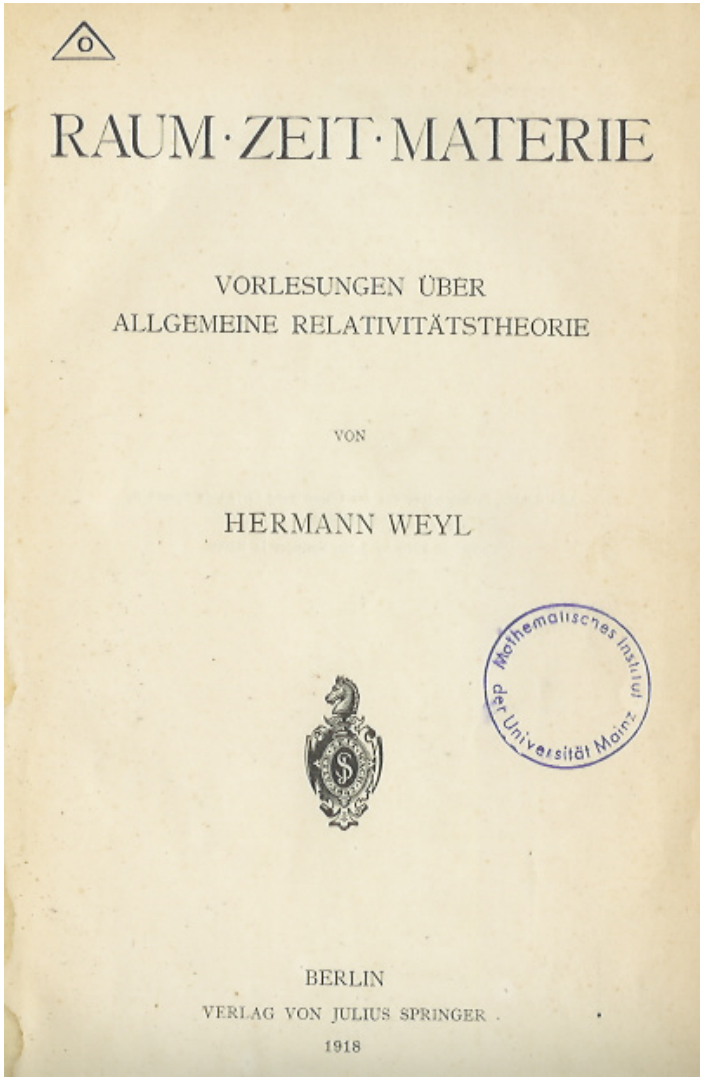
De Sitter's Spacetime as a Threat to Mach's Principle

Einstein (1918): "If the De Sitter solution were valid everywhere, it would show that the introduction of the 'λ-term' does not fulfill the purpose I intended. Because, in my opinion, **the general theory of relativity is a satisfying system only if it shows that the physical qualities of space are *completely* determined by matter alone. Therefore, no $g_{\mu\nu}$-field can exist (that is, no spacetime continuum is possible) without matter that generates it."**

Einstein's Critique

- Einstein noted that the metric tensor vanishes on the horizon, and then concluded:
- **"The De Sitter system does not look at all like a world free of matter, but rather like a world whose matter is concentrated entirely on the [horizon] surface.** This could possibly be demonstrated by means of a limiting process . . .."
- Not long afterward **Hermann Weyl would provide the details for this argument.**

Weyl's *Raum-Zeit-Materie*, First Edition, 1918



RAUM·ZEIT·MATERIE

VORLESUNGEN ÜBER
ALLGEMEINE RELATIVITÄTSTHEORIE

VON

HERMANN WEYL

BERLIN
VERLAG VON JULIUS SPRINGER
1918

Hubert Goenner on Weyl's Shifting Views on de Sitter Spacetime

Hubert Goenner, "Weyl's Contributions to Cosmology," in Erhard Scholz, ed., *Hermann Weyl's Raum – Zeit – Materie and a General Introduction to his Scientific Work,* DMV Seminar Band 30, Birkhäuser, 2001

How Felix Klein used Projective Geometry to Clarify the Issue of Singularities

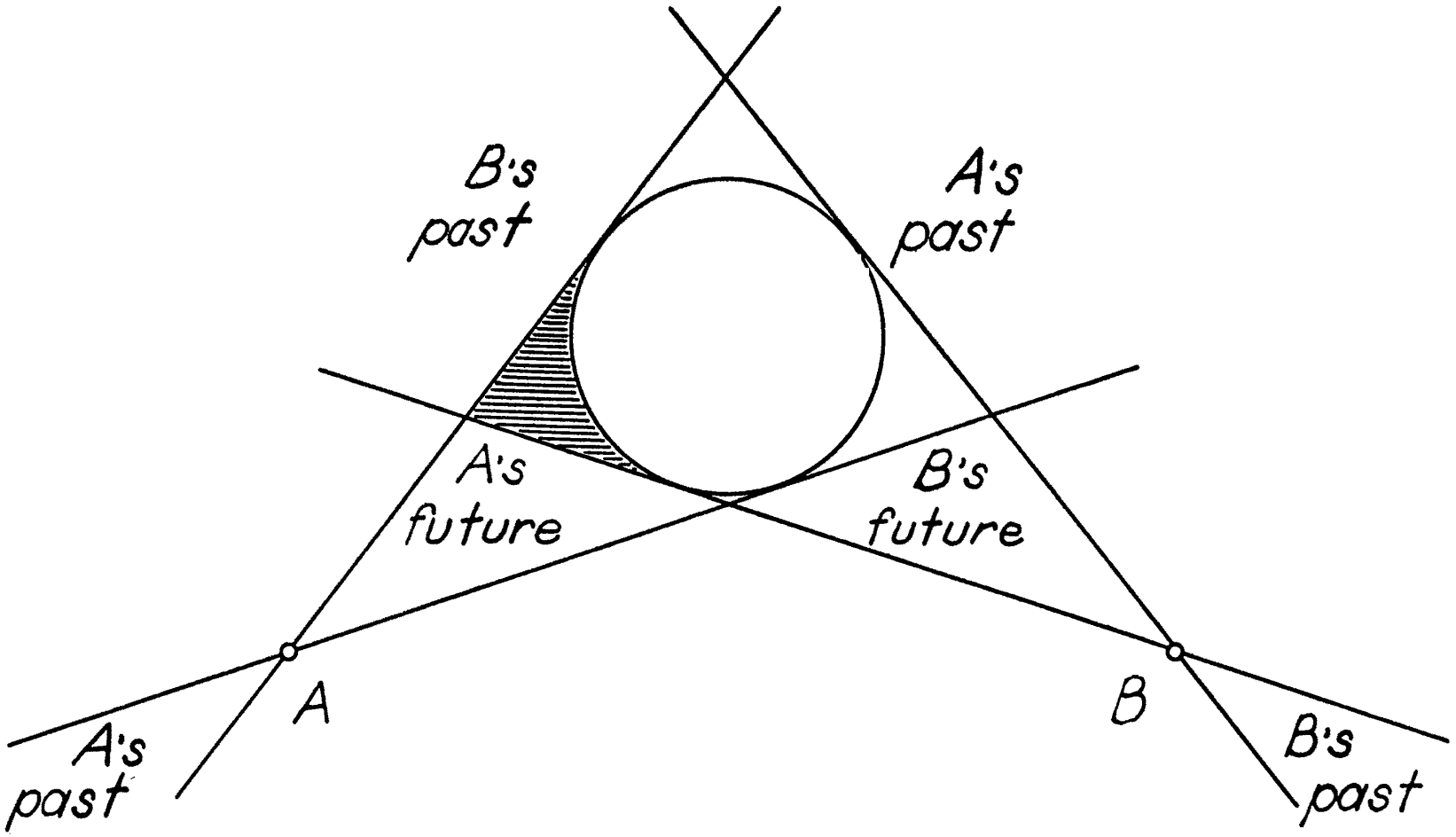
Klein to Einstein, 14 June 1918

Klein reacted to Einstein's critique of de Sitter's model: **"I came to the conclusion that the singularity you noticed can simply be transformed away."**

Klein considered pencils of hyperplanes intersecting the **quadric at temporal infinity**. This yields a 6-parameter transitive group mapping the spacetime to itself.

But observers in different worlds experience strange difficulties when they try to communicate with one another.

How Can 2 Observers Synchronize Watches in de Sitter Spacetime?



Klein to Einstein, 14 June 1918

"It is amusing to picture how two observers living on the quasi-sphere with differing de Sitter clocks would squabble with each other. Each of them would assign finite ordinates to some of the events that for the other would be lying at infinity or that would even show imaginary time values."

Einstein to Klein, 20 June 1918

"You are entirely right. De Sitter's world is, in and of itself, free of singularities and its spacetime points are all equivalent. . . . **My critical remark about de Sitter's solution needs correction: a singularity-free solution for the gravitational field equations without matter does in fact exist.** However, under no condition could this world come into consideration as a physical possibility."

Why didn't this Concession lead to a Retraction or Modification of Einstein's Critique of de Sitter?

See Michel Janssen, Editorial Note on the Einstein – de Sitter Debate, *CPAE,* vol. 8

Princeton Lectures, 1922

- Einstein argues just as he did in 1917: he makes **no mention of de Sitter ‘s alternative solution!**
- He discusses the results of **Thirring and Lense** in much the same way as when he discussed Machian effects back in 1912-13
- He also notes the **purely theoretical** character of such **induction effects from frame dragging**: but as before, these were way **too small to be tested experimentally**

Space must be closed and finite

- „If the universe were quasi-Euclidean, then Mach was wholly wrong in thinking that inertia, as well as gravitation, depends upon a kind of mutual interaction between bodies.“
- **But what about a universe that is closed, finite, and (essentially) matter free?**
- Einstein simply ignored that **his cosmological field equations admit a solution at odds with Machian principles**

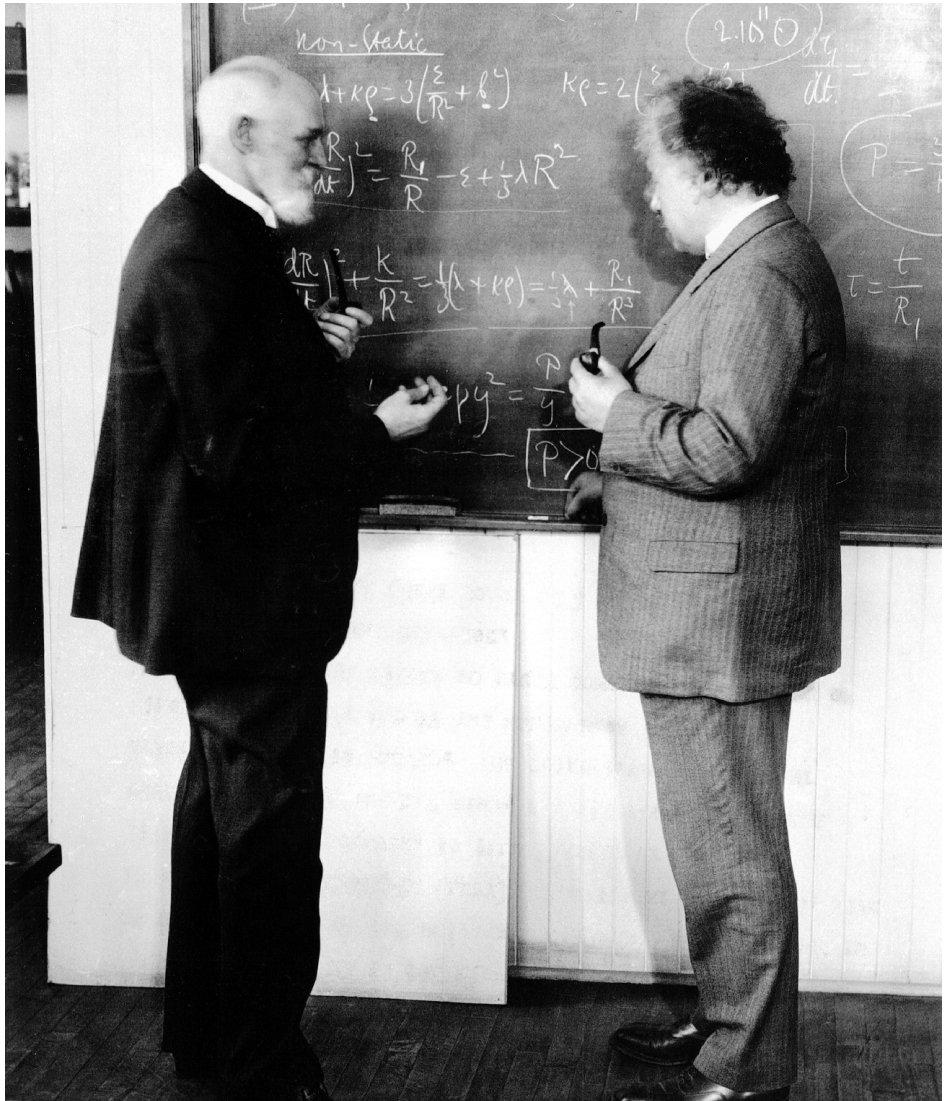
Weyl ‘s fifth edition of *RZM*, 1923

- Weyl was a key intermediate figure in the debate between Einstein and de Sitter
- At first he sided very strongly with Einstein
- But **in 1924 Weyl broke with Einstein’s “Relativity Church” over the Dogma of Mach’s Principle**
- That change of heart was made public in „Massenträgheit und Kosmos: Ein Dialog“, *Die Naturwissenschaften*

Einstein and de Sitter sign a Truce and abandon „Static Models“

Caltech in 1932

The Einstein- de Sitter Model, 1932



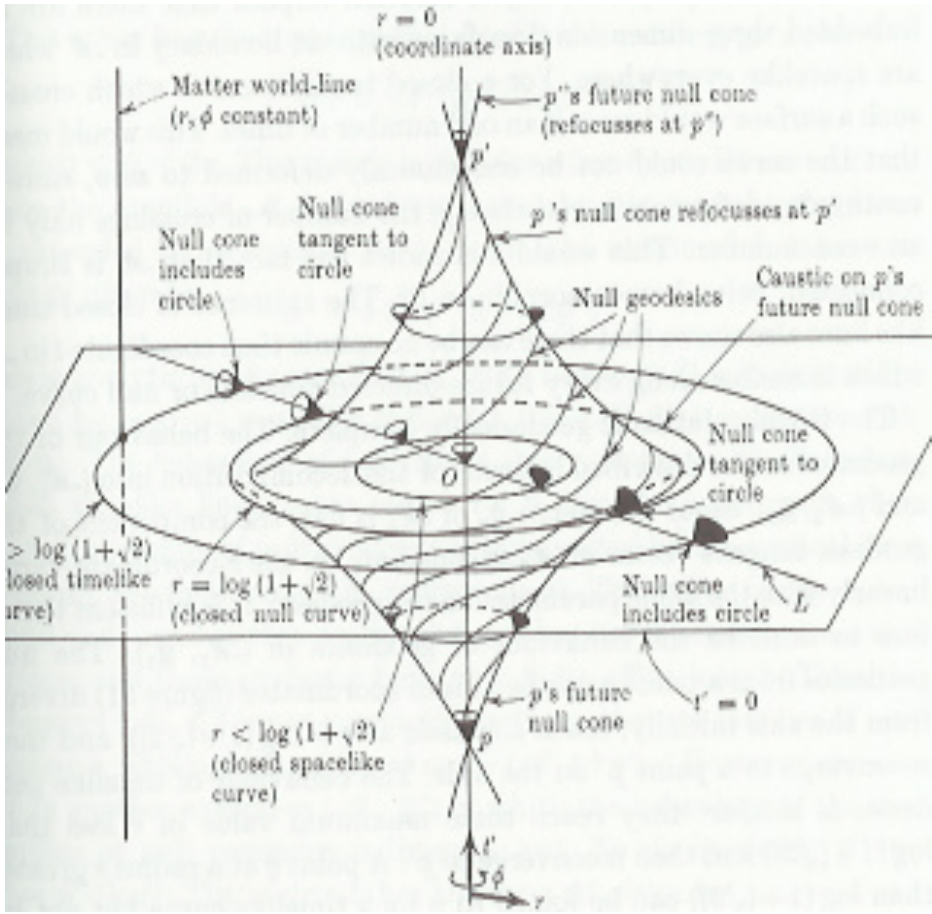
Howard Percy „Bob“ Robertson, an early cosmological theorist

- In 1929 he showed that the **Einstein and de Sitter worlds** were the only **stationary cosmologies** that were **spatially homogeneous and isotropic** (within the class of all **Robertson-Walker spacetimes**)
- In 1933 he introduced the terminology of **cosmological models**

Celebrating Einstein's 70th Birthday at Princeton

Gödel's Birthday Present: an anti-Machian Spacetime

- 1916: Einstein pleaded with Carathéodory to take up the **Problem of Closed Timelike Curves**
- 1949: Gödel's Model displays this property
- **Cosmologies with rotation**: anti-Machian models (Oszvath and Schücking, 1960s)



Einstein repudiates Mach's Principle (1949)

„. . . for a long time I considered [Mach's conception] as, in principle, the correct one. It presupposes implicitly, however, that **the basic theory should be of the general type of Newton's mechanics** . . .. The attempt at such a solution **does not fit into a consistent field theory**, as will be immediately recognized." (*"Autobiographical Notes,"* 1949)

Einstein to Felix Pirani, 2 February 1954

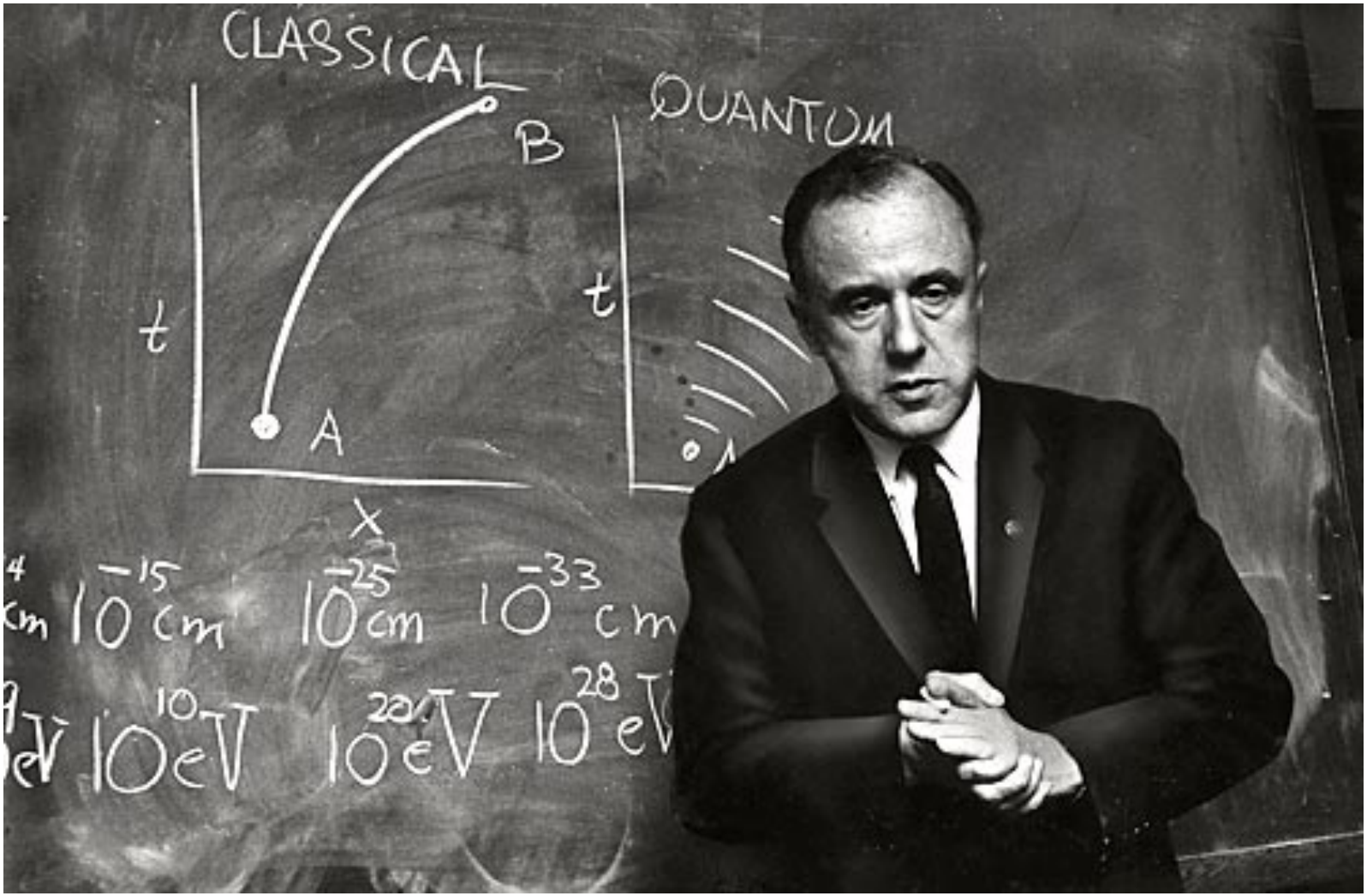
„Von dem Machschen Prinzip sollte man eigentlich überhaupt nicht mehr sprechen.“

Pirani ignored this advance and made Mach ‘s Principle the topic for his talk at the Bern Jubilee of Relativity Theory in July 1955

Pirani at the Bern Jubilee, 1955

- Pirani distinguished **four different versions of Mach ‘s Principle**
- The second of these was **Einstein ‘s version**: „The gravitational field (metric tensor) is **determined** by the material content of space-time (energy-momentum tensor).“
- Pirani noted that the work of **Lichnerowicz** and his school had shown **the importance of boundary conditions** for solving Einstein ‘s field equations. Could one do without them?

Wheeler on Machian Spacetimes



Cosmological Models at the Bern Jubilee, 1955

1) H. P. Robertson (Pasadena and Paris): „Cosmological Theory“

2) M. von Laue (Berlin): „Zur Kosmologie“

3) J. Tits (Bruxelles): „Espaces homogènes et isotropes de la Relativité“

4) O. Heckmann u. E. Schücking (Hamburg): „Ein Weltmodell der Newtonschen Kosmologie mit Expansion und Rotation“

Otto Heckmann and Engelbert Schücking

H. P. Robertson on „Cosmological Theory“ (1955)

„A number of field equations which warrant attention have been proposed, including those . . . by **Hoyle and Jordan**, and we may well expect that a fruitful interaction between the relativity and quantum theories will produce others more inclusive in the near future. Nevertheless I shall confine myself . . . to **those models . . . governed by the field equations of the general theory of relativity** –“

H. P. Robertson on „Cosmological Theory“ (1955)

„which **in spite of its impotence in dealing with Mach ‘s Principle or the microscopic realm** may yet be the springboard from which a more complete theory takes off, much as it itself took off from the Newtonian theory.“

Discussion Followed . . .

O. Heckmann: The existence of Gödel ‘s solution proves that there exists an ‚absolute‘ rotation in the theory of relativity.

H. P. Robertson: I am afraid that is correct. The entire material field in his solution must be judged to be in rotation. I consider it a defect in the field equations of the general theory of relativity that they allow such a solution.

Discussion Followed . . .

J. Ehlers: [from your assumptions] you get only those models which have a cosmic time-coordinate . . .. Are there arguments by which it is possible to exclude such models with an intrinsic rotation in which it is impossible to have such a time-coordinate?

H. P. Robertson: I am not aware of any argument which could enable one to exclude such models *a priori.*

„An Anti-Mach Metric,“ Istvan Oszváth and Engelbert Schücking (1962)

Refuting Pirani ‘s

„Mach-3“ Principle

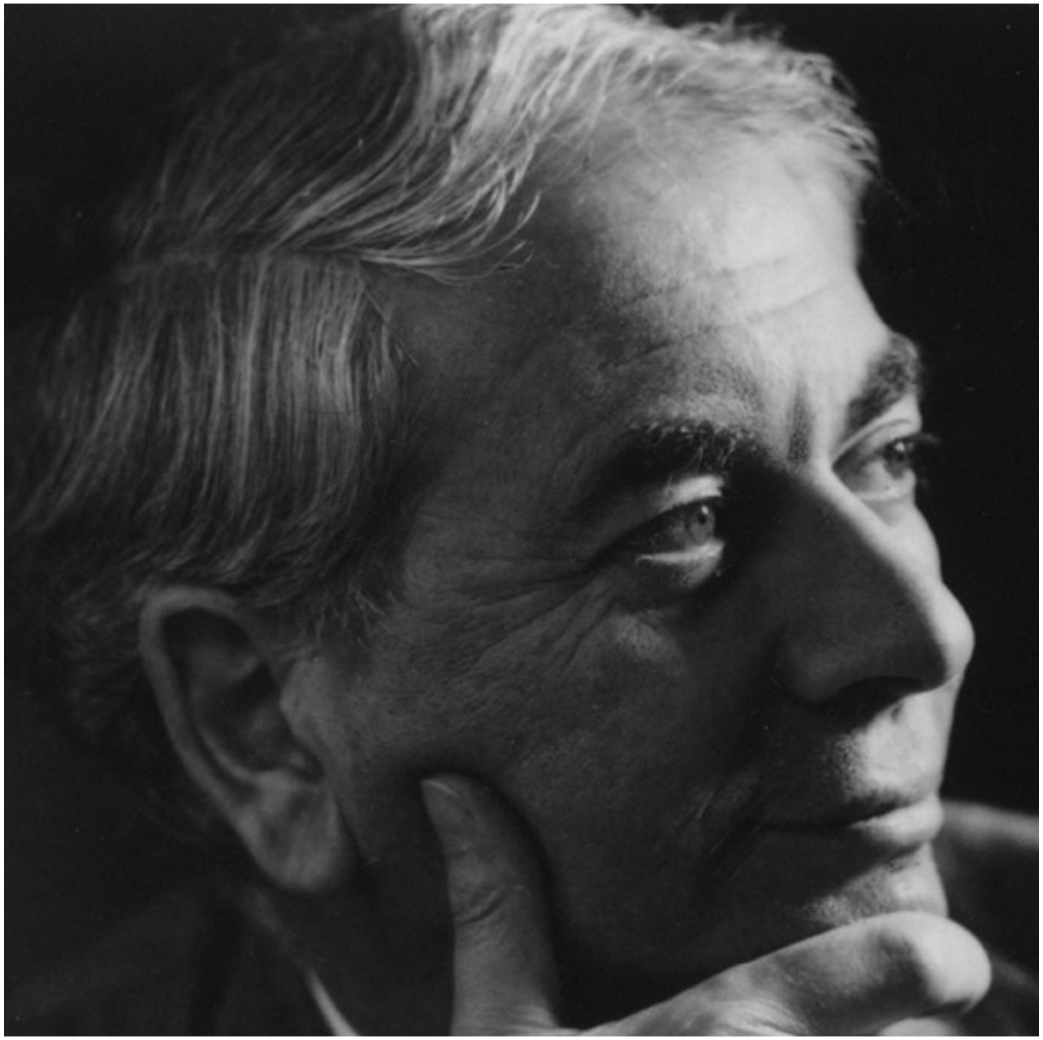
Oszváth and Schücking (1962)

„**Abstract.** An exact solution is given for Einstein's vacuum field equations which is free of singularities, is complete, and has a nonvanishing Riemann tensor. The curvature tensor of this anti-Mach-metric is of the null-type in the Petrov-Pirani-Penrose classification. . .. “

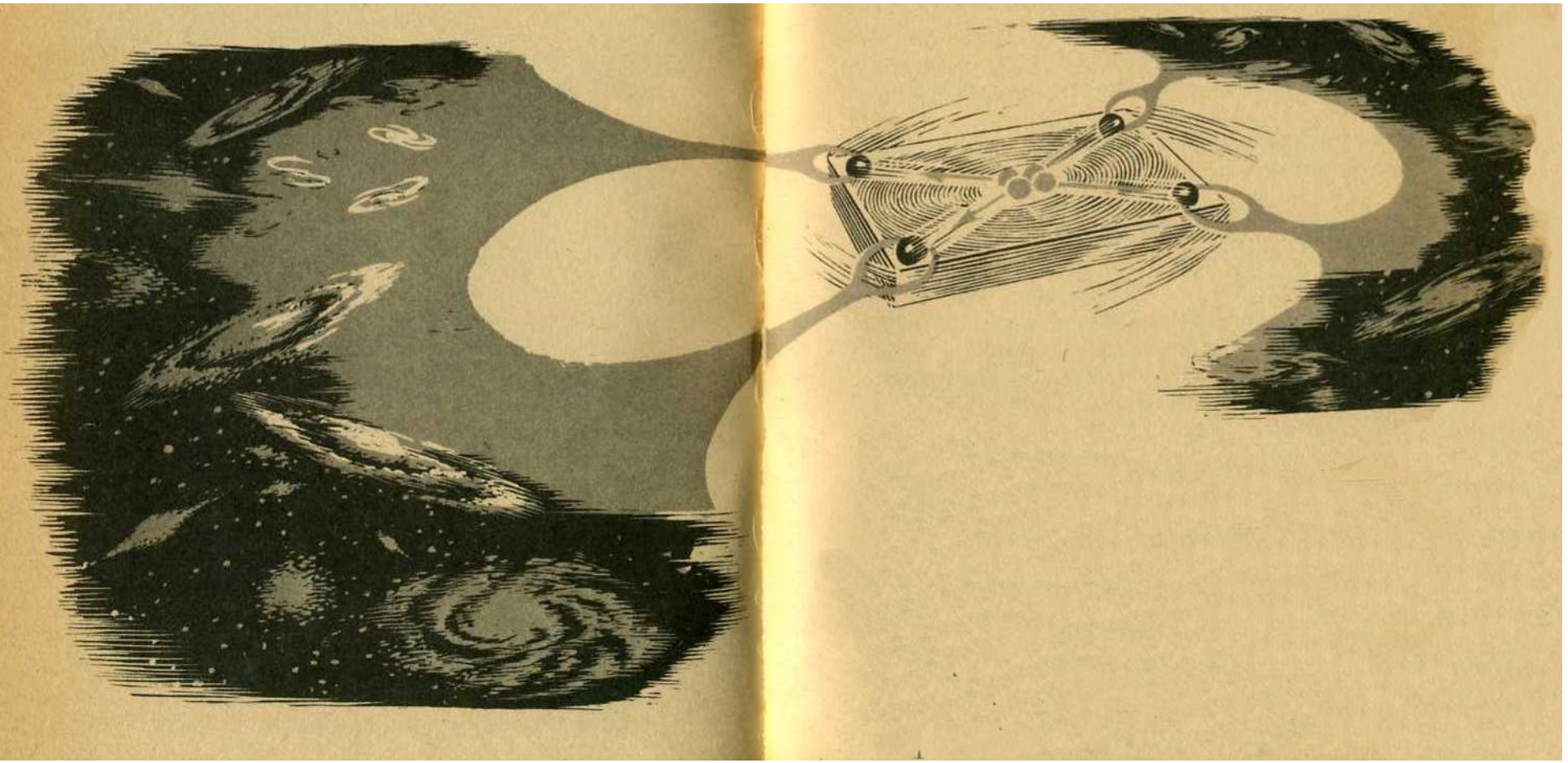
Oszváth and Schücking (1962)

- **Pirani ‘s Mach-3**: „In the absence of matter, space-time should necessarily be Minkowskian.“
- Oszváth and Schücking **interpret „the absence of matter“ as absence of singularities**.
- They then give a non-singular solution of Einstein ‘s vacuum field equations with nonvanishing Riemann tensor.
- So **this space-time cannot be Minkowskian**.

On the Views of an Influential Machian Cosmologist



Sciama's Machian Universe (from Martin Gardner, *The Relativity Explosion)*



Dennis Sciama AHQP Interview, 1975

. . . Sometimes I've had a preference for the universe that just expands forever with the velocity tending to zero, the **Einstein-de Sitter model.**

I once hoped, but that hope has not been realized in our work up to date, that **Mach's Principle would lead to the unique model** of the Universe, and then the hope was that it would be the one I've just described.

Dennis Sciama AHQP Interview, 1975

. . . Well, if you think that kinetic energy manifesting inertia is due to gravitation, then **you might intuit that the most Machian way of having one made by the other would be if there's an equal amount of energy, which would give you uniquely the Einstein-de Sitter model.** I still have a secret hope that that might turn out so, but it may well not. . . .

Dennis Sciama AHQP Interview, 1975

But the work we've done so far has shown that **all** these **highly symmetrical models**, the **Robertson-Walker models**, **seem to be Machian**, which may be a weakness of our Mach Principle still.

Engelbert Schücking, speaking for the skeptics

Engelbert Schücking on Machian Effects

"Mach's principles – whatever they may be – will always find their defenders and believers. When one of its promoters, Dennis Sciama, slammed on the brakes of his car, propelling his girlfriend, seated next to him, toward the windshield, she was heard to be moaning,

'All those distant galaxies!'"